群表示与相关数学问题研讨会

程序册

主办单位

天元数学国际交流中心

2025.06.29—07.05

中国 • 昆明

会议信息

天元数学国际交流中心 2025 年“群表示与相关数学问题”研讨会，会议时间 2025 年 6 月 29 日—2025 年 7 月 5 日，会议地点：天元数学国际交流中心，中心位于云南省昆明市宜良县柴石滩水库库区。会议将邀请国内外表示论相关专家学者，聚焦有限群及其表示论领域 Alperin 权猜想、Broué 导等价猜想等广受关注的局部–整体猜想前沿焦点问题，报告相关数学问题前沿研究动态，研讨交流 Alperin 权猜想等问题的最新研究进展，促进表示论相关领域的交叉与融合。开设表示论相关专题短期课程，邀请表示论领域的权威专家授课，为有志于此方向的年轻学者提供高水平学习和交流的平台，为将来从事相关领域的学习和研究奠定坚实的基础。

学术指导: 张继平院士

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会议地点: 天元数学国际交流中心

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08:30-09:30			余君	冯致程	陈亦飞	王立中
09:30-10:30			陈亦飞	王立中	徐行忠	刘燕俊
10:30-10:50			茶歇			
10:50-11:30			邱宇	王立中	陈哲	刘燕俊
			午餐			
14:30-16:00			Olivier Dudas	张继平	Olivier Dudas	张继平
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17:15-18:15			陈亦飞	王立中	徐行忠	刘燕俊
			晚餐			
20:00-21:30	Olivier Dudas	张继平				
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09:50-10:10	茶歇						
10:10-10:50 10:50-11:30	徐斐 李利平	周远扬		杨丹丹 栾永志	王宝山	胡学琴 黄鑫	廖军
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二　会议日程

6 月 29 日（星期日）：报到			
时间	报告人	题目	主持人
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09:30–10:30	陈亦飞	An introduction to McKay correspondence I	王立中
10:30–10:50	茶歇		
10:50–11:30	邱宇	From green mutation to X-evolution on cluster complexes	王立中
11:30–14:30	午餐		
14:30–16:00	Olivier Dudas	Symmetries of characters of finite reductive groups II	张继平
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16:15–17:15	余君	Finite order automorphisms of Lie algbras and algebraic groups II	冯致程
17:15–18:15	陈亦飞	An introduction to McKay correspondence II	王立中
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19:30–20:30	徐行忠	谱序列 I	刘燕俊

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10:50–11:30	陈哲	On Deligne–Lusztig theory over cdvr: algebraisation, dimension, and sign	刘燕俊
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14:30–16:00	Olivier Dudas	Symmetries of characters of finite reductive groups III	张继平
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09:10–09:50	吴泉水	Numerical Homological Regularities	
09:50–10:10	茶歇		
10:10–10:50	徐斐	关于群与范畴的 2-表示	周远扬
10:50–11:30	李利平	Continuous representations of infinite permutation groups and applications	
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14:00–14:40	黄华林	从有限群到有限张量范畴	李才恒
14:40–15:20	熊欢	Equidistributed statistics on integer partitions with fixed perimeter	
15:20–16:00	李从辉	On fusion systems of blocks of finite groups and stable equivalences of Morita type	
16:00–16:20	茶歇		
16:20–17:00	陈刚	Schur rings over infinite groups	张红莲
17:00–17:40	杜予聪	The blockwise Galois Alperin weight conjecture for symmetric and alternating groups and their double covering groups	
17:40–18:20	孙嘉庆	McKay graphs and Euclidean diagrams	

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09:10–09:50	陈佳源	Schur-Weyl duality between $GL(n, Q_p)$ and $GL(m, C)$	
09:50–10:10	茶歇		
10:10–10:50	杨丹丹	The homomorphism-homogeneity of algebraic structures	王宝山
10:50–11:30	栾永志	Study on the Clifford group	
11:30–14:30	午餐		
14:30–15:10	钱国华	特征标的余次数图	刘合国
15:10–15:50	颜全福	On the number of prime divisors counting multiplicities of character codegrees	
15:50–16:10	茶歇		
16:10–16:50	杨进伟	Representation and tensor category of affine sl_2 Lie algebra	王立中
16:50–17:30	林兴君	Automorphism groups of vertex operator algebras	
17:30–18:10	余铌娜	Zhu Algebras of Permutation Orbifold Vertex Operator Algebras	

7 月 4 日（星期五）：学术活动			
时间	报告人	题目	主持人
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09:10–09:50	周胜林	Reduction for flag-transitive symmetric 2-designs with $\gcd(k,\lambda)=p$	
09:50–10:10	茶歇		
10:10–10:50	胡学琴	Hyperfocal subalgebras of blocks with Frobenius hyperfocal inertial quotients	廖军
10:50–11:30	黄鑫	On Broué's abelian defect group conjecture and Brauer character bijections	
11:30–14:30	午餐		
14:30–15:10	娄本功	Hexavalent edge-transitive Cayley graphs on nonabelian simple groups	海进科
15:10–15:50	范文文	Regular dessins with underlying graph $K_{n,p}$	
15:50–16:10	茶歇		
16:10–16:50	吴辞旋	Non-solvable covers of 2-arc-transitive graphs	刘伟俊
16:50–17:30	王磊	Odd power word maps	
17:30–18:10	刘洋	Finite groups with few character codegrees	

7 月 5 日（星期六）：自由讨论交流，离会

三　短期课程摘要

An introduction to McKay correspondence

陈亦飞 (中科院数学与系统科学研究院)

这个短课程将介绍经典的 McKay 对应，即，SL2(C) 有限子群 G 的不可约表示与曲面 ADE 型奇点 (即 C2/G 出现的奇点) 极小解消后的例外曲线的对应关系。该课程将介绍 McKay 对应中涉及到的代数几何知识。大致分为代数簇、商簇、奇点和奇点解消，曲面相交数，不变量理论。时间允许的话，还会介绍几何 McKay 对应，即 G 的非平凡不可约表示与 C2/G 的极小解消后的例外曲线的一一对应关系。

参考文献：

1) I Dolgachev, McKay Correspondence, 2009

2) I Shafarevich, Basic Algebraic Geometry

Symmetries of characters of finite reductive groups (online)

Olivier Dudas(*Aix-Marseille University*)

The space of class functions on a reductive group over a finite field (such as GL(n,q), Sp(2n,q), etc.) admits two particularly interesting bases:

- an algebraic basis, given by the characters of irreducible representations,

- a geometric basis, given by the characteristic functions of character sheaves.

In this series of lectures, we will explain how these two bases are related. It involves a transformation that generalizes the classical Fourier transform on finite abelian groups, which was discovered by Lusztig when classifying the irreducible characters.

We will provide a new approach to understand this transformation, using traces of braid group operators acting on Deligne – Lusztig varieties. This viewpoint leads to a natural SL2(Z)-action on the space of class functions, bringing together the Fourier transform, the Frobenius eigenvalues on the cohomology of Deligne–Lusztig varieties, and Shintani's twisting operator, which exchanges the Frobenius morphism with its inverse.

If time permits, we will also discuss applications to the theory of Spetses —conjectural generalizations of finite reductive groups whose combinatorics are governed by complex reflection groups rather than Weyl groups. In this setting, we propose natural candidates for Fourier matrices and unipotent character sheaves.

(This is a work in progress with Bonnafé-Broué-Malle-Michel-Rouquier.)

谱序列

徐行忠 (湖北大学)

Spectral Sequences(谱序列) 是计算同调与上同调的一个主要工具。我们主要介绍谱序列的基础知识以及一些应用。主要参考书籍是

1. K. Brown, Cohomology of Groups, Graduate Texts in Mathematics 87, Springer-Verlag, New York, 1982. (第七章)

2. J. Rotman, An Introduction to Homological Algebra, Universitext, 2009. (第十章)

Finite order automorphisms of Lie algbras and algebraic groups

余　君 (北京大学)

We present the theory of Kac coordinates of finite automorphisms of complex semsimple Lie algebras. As an application, we classify small order automorphisms of complex simple Lie groups and finite reductive Lie groups.

四　会议报告摘要

Schur rings over infinite groups

陈　刚 (海南大学)

In this talk, Schur rings over several classes of infinite groups will be classified.

Schur-Weyl duality between $GL(n, Q_p)$ and $GL(m, C)$

陈佳源 (香港大学)

In a recent joint work with Daniel Wong (CUHK (Shenzhen)), we construct a Schur-Weyl duality between representations of $GL(n, Q_p)$ and $GL(m, C)$. This functor has several nice properties such as preserving irreducibility and parabolic induction, and relating Bernstein-Zelevinsky derivatives and tensor product problems. This talk will discuss some applications and some further properties of such duality and compare with recent geometric study of Deng-Huang-Xu-Zhao.

On Deligne–Lusztig theory over cdvr: algebraisation, dimension, and sign

陈　哲 (汕头大学)

In this talk we discuss an algebraisation of generic Deligne–Lusztig representations of connected reductive groups over a cdvr, proved in two joint works with Stasinski. This is an explicit character formula and provides an affirmative answer to a question raised by Lusztig. We will start with the special case of GL_n, illustrating the situation using orbits, and then explain how to pass from GL_n to arbitrary reductive groups. In the end, we present a conjectural sign formula, which is unconditional on all parameters.

The blockwise Galois Alperin weight conjecture for symmetric and alternating groups and their double covering groups

杜予聪 (重庆大学)

The Galois Alperin Weight Conjecture is one of the most important global-local conjectures in modular representation theory of finite groups. It implies a relation between irreducible representations of a finite group and irreducible projective representations of quotient groups of its local subgroups. This conjecture has been reduced to inductive conditions of finite simple groups, that is, to prove this conjecture, it suffices to prove the inductive Galois Alperin Weight Conditions for finite simple groups. In this lecture, we prove the blockwise Galois Alperin Weight Conjecture for symmetric and alternating groups and their double covering groups. This is a joint work with Xin Huang and Jiping Zhang.

Regular dessins with underlying graph $K_{n,p}$

范文文 (云南师范大学)

A complete dessin is an orientable map with underlying graph being a complete bipartite graph, which is said to be regular if all edges are equivalent under the group of color- and orientation- preserving automorphisms, and circular if the boundary cycle of each face is a circuit (a simple cycle). As one of a series papers towards a classification of complete regular dessins, this paper we classify and enumerate regular dessins and circular regular dessins of $K_{n,p}$ with p a prime and $n \geq 2$ being an integer.

Hyperfocal subalgebras of blocks with Frobenius hyperfocal inertial quotients

胡学琴 (华中师范大学)

In this talk, we first introduce the definition of a hyperfocal inertial quotient which is an analogue of an inertial quotient at the hyperfocal level. By describing the structure of this quotient group, we can define the blocks with Frobenius hyperfocal inertial quotients. These types of blocks include blocks with cyclic hyperfocal subgroups and blocks with

Klein four hyperfocal subgroups these two classical types. Then we will investigate the structure of hyperfocal subalgebras of these blocks in terms of the stable category. As applications, we can get the structures of the derived categories of hyperfocal subalgebras with cyclic hyperfocal subgroups and Klein four hyperfocal subgroups and then we can verify Broué's abelian defect group conjecture for blocks with Klein four hyperfocal subgroups. This is a joint work with Yuanyang Zhou.

从有限群到有限张量范畴

黄华林 (华侨大学)

本报告旨在阐述如何运用有限群的表示论和上同调等工具来研究点化有限张量范畴的分类理论。我们将聚焦于交换群上点化有限张量范畴的研究进展，并探讨该领域当前面临的主要挑战。报告内容主要基于报告人与刘公祥、杨毓萍及叶郁合作的研究成果。

On Broué's abelian defect group conjecture and Brauer character bijections

黄　鑫 (华中师范大学)

For blocks of finite groups with abelian defect groups, it is well-known that Broué's abelian defect group conjecture implies the Alperin-McKay conjecture and the Alperin weight conjecture. In 2004, Navarro proposed refined versions of the Alperin-McKay conjecture and the Alperin weight conjecture by taking certain Galois automorphisms into consideration. In 2018, Kessar and Linckelmann proposed a refined version of Broué's conjecture, and they showed that the refined Broué conjecture implies the Galois-Alperin-McKay conjecture (for blocks with abelian defect groups). In this talk we show that the refined Broué conjecture implies the Galois Alperin weight conjecture (of course, for blocks with abelian defect groups). By the way, we report some progress on the refined abelian defect group conjecture and the Galois Alperin weight conjecture.

On fusion systems of blocks of finite groups and stable equivalences of Morita type

李从辉 (西南交通大学)

Fusion systems is a method to organize local data of blocks. Stable equivalences of Morita type is a useful tool in the study of blocks of finite groups. In this talk, we share an observation that stable equivalences of Morita type between blocks induce isomorphisms of some quotient fusion systems.

Continuous representations of infinite permutation groups and applications

李利平 (湖南师范大学)

Let Ω be an infinite set and G an infinite group acting on it. In this talk I will describe continuous representations of G and the following applications in commutative algebra, geometric group theory, and representation theory of categories: linear stable range of homology groups of congruence subgroups of general linear groups, symmetric notherianity of polynomial rings, a classification of Grothendieck topologies on the orbit category, and etc.

Automorphism groups of vertex operator algebras

林兴君 (武汉大学)

In the theory of vertex operator algebras, it is an important problem to determine automorphism groups of vertex operator algebras. For instance, the automorphism group of the moonshine vertex operator algebra is isomorphic to the Moster group. We will discuss automorphism groups of parafermion vertex operator algebras. We will also discuss fusion rules about twisted modules, which are closely related to automorphism groups.

Finite groups with few character codegrees

刘　洋 (天津师范大学)

Let G be a finite group and χ be an irreducible complex character of G. We call $\mathrm{cod}(\chi) = |G : ker(\chi)|/\chi(1)$ the codegree of χ. In this talk, I will introduce some results on finite groups with few character codegrees.

Hexavalent edge-transitive Cayley graphs on nonabelian simple groups

娄本功 (云南大学)

Let Γ be a $2p$-valent edge-transitive Cayley graph on a nonabelian simple group G, where p is a prime. For $p = 2$, Fang et al. [Europ. J. Combin. 25 (2004), 1107–1116] characterized such nonnormal graphs Γ and developed construction methods for Cayley graphs with specific symmetric properties. For $p = 3$, Pan et al.[Comm. Algebra 50(2022), 4891–4905] classified all nonnormal 2-arc-transitive Cayley graphs in this setting. Building on these foundational results, this talk focuses on hexavalent edge-transitive but not 2-arc-transitive Cayley graphs. We determine all candidate nonabelian simple groups G admitting such nonnormal edge-transitive but not 2-arc-transitive Cayley graphs. This is the joint work with Yulong Ma.

Study on the Clifford group

栾永志 (山东大学)

Given an orthonormal basis $\{e_1, e_2, \ldots, e_n\}$ of $\mathbb{R}^n$. A **Clifford group** Γ_n is a finite group which is generated by $\{e_1, e_2, \ldots, e_n\}$. It can be presented by the abstract elements $\{e_1, e_2, \ldots, e_n, -1\}$ subject to the relations that -1 is central and that $(-1)^2 = 1$, $e_j^2 = -1$, $\forall\, j = 1, 2, \ldots, n$, and $e_j e_k = -e_k e_j$, $\forall 1 \leq j \neq k \leq n$. We study the automorphism group and some generalizations of Γ_n.

An identity basis for the minimal non-finitely based involution semigroup $(A_0^1, ^*)$

罗彦锋 (兰州大学)

Recently, we proved that the five-element monoid $A_0^1 = \langle \mathrm{e}, \mathrm{f} \mid \mathrm{e}^2 = \mathrm{e}, \mathrm{f}^2 = \mathrm{f}, \mathrm{ef} = 0 \rangle \cup \{1\}$ with the unique involution operation * that interchanges e and f and fixes all other elements is the unique minimal non-finitely based involution semigroup. In this talk, we give an infinite identity basis for $(A_0^1, ^*)$ and show that it generates a variety whose finite membership problem admits a polynomial algorithm.

特征标的余次数图

钱国华 (苏州工学院)

我们将介绍关于特征标余次数图方面的若干工作.

From green mutation to X-evolution on cluster complexes

邱　宇 (清华大学)

We introduce flows and foliations on cluster complexes, generalizing green mutation on cluster exchange graphs, with application to the topology of cluster complexes.This is based on arXiv: 2501.15756 with Tang Liheng.

McKay graphs and Euclidean diagrams

孙嘉庆 (北京大学)

This talk investigates the representation theory of a finite group G over a field k (whose characteristic does not divide $|G|$), with a focus on the McKay quiver and its generalizations associated with a 2-dimensional kG-module V. Let $\{V_1, \ldots, V_n\}$ be denote complete set of representatives of isomorphism classes of simple (left) kG-modules. By decomposing $V \otimes_k V_i$ and $V^* \otimes_k V_i$ into direct sums of simple modules, we can define a valued quiver $\mathrm{M}(V, V^*)$.

When k is algebraically closed, Auslander and Reiten showed that the underlying graph of the separated quiver is a disjoint union of extended Dynkin diagrams $\widetilde{A}_n$, $\widetilde{D}_n$, $\widetilde{E}_6$, $\widetilde{E}_7$, $\widetilde{E}_8$. We generalize this result to arbitrary fields k, proving that the underlying valued graph of the separated quiver remains a union of Euclidean diagrams. Based on

this, we further classify the possible decompositions of $V \otimes_k V_i$ into 11 distinct cases.

Odd power word maps

王　磊 (云南大学)

With two exceptions, we prove surjectivity of odd power word maps on finite non-abelian simple groups. More precisely, we prove the following: if N is any odd integer, then the word map $(x, y) \mapsto x^N y^N$ is surjective on every finite non-abelian simple group except $\mathrm{PSL}_2(q)$ and ${}^2G_2(q)$.

Numerical Homological Regularities

吴泉水 (复旦大学)

Inspired by the studies in algebraic geometry and commutative algebra, Jorgensen defined Castelunovo-Mumford regularities for graded modules over noncommutative noetherian connected graded algebras. I will survey new developments in recent years and our recent work on the numerical homological regularities with Bojuan Yi.

Non-solvable covers of 2-arc-transitive graphs

吴辞旋 (云南财经大学)

In this talk, we will report on the second step of Praeger's program on 2-arc-transitive graphs. We present constructions of connected 2-arc-transitive covers of the 2-arc-transitive Petersen graph and 2-arc-transitive complete graphs with non-solvable automorphism groups. To the best of our knowledge, these are the first known examples of such covers. This is joint work with Jiyong Chen, Cai Heng Li and Yan Zhou Zhu.

Some problems in representations of algebras

惠昌常 (首都师范大学)

In this talk I will survey several major conjectures in representation theory and homological algebras of Artin algebras. Especially we talk about the Nakayama and finitistic dimension conjectures and some advances on them.

Equidistributed statistics on integer partitions with fixed perimeter

熊　欢 (哈尔滨工业大学)

Compared to permutation statistics, the equidistribution phenomenon for statistics on integer partitions turns out to be very rare. In this talk, I will introduce our results on the equidistributed statistics of integer partitions with fixed perimeter. We proved that the number of even parts and the count of repeated parts have the same distribution over integer partitions with a fixed perimeter. This refines Straub's analog of Euler's odd-distinct partition theorem. We generalize the two concerned statistics to those of the part-difference less than d and the parts not congruent to 1 modulo $d+1$, and prove a distribution inequality over partitions with a prescribed perimeter. Both of our results are proven analytically and combinatorially. This is a joint work with Zhicong Lin and Sherry H.F. Yan.

关于群与范畴的 2-表示

徐　斐 (汕头大学)

基于 2-范畴理论，Ganter 和 Kapranov 引入了有限群的 2-表示理论，而 Balmer 也引入了群表示叠形概念。以上工作均围绕从 1-范畴到 2-范畴的伪函子开展研究。我们回顾上述研究思路，随后在此基础上考虑有限范畴的 2-表示，从一个具体案例揭示 2-表示理论与代数表示论、层论的新关联。

On the number of prime divisors counting multiplicities of character codegrees

颜全福 (北京大学)

In this presentation, we introduce a further contribution to the topic of character codegrees of finite groups. We denote the number of prime divisors (counting multiplicities) of n by $w(n)$. Let $\mathrm{Cod}(G)$ be the set of irreducible character codegrees of a finite group G. We introduce a new arithmetical concept concerning character codegrees: $w_c(G) = \max_{d\in \mathrm{Cod}(G)} w(d)$. First, we study the structure of groups G for which $w_c(G)$ is small. On the other hand, we investigate the relationship between $w_c(G)$ and $|\pi(G)|$, the number of distinct prime divisors of the order of G, and we show that $|\pi(G)| \leq 6w_c(G)$. In particular, we obtain a sharp bound in the cases where $w_c(G) = 2, 3$ or 4, as well as in the case when G is supersolvable.

The homomorphism-homogeneity of algebraic structures

杨丹丹 (西安电子科技大学)

A countable structure A is said to be *homomorphism-homogeneous* if every homomorphism between its finitely generated substructures extends to an endomorphism of A. In this talk, I will give a complete and explicit classification of homomorphism-homogeneous finite semilattices, addressing an open problem posed by Dolinka and Mašulović in 2011. This is joint work with W. Yang.

Representation and tensor category of affine sl_2 Lie algebra

杨进伟 (上海交通大学)

In this talk, I will give a summary on representations and tensor categories of affine sl_2 Lie algebras, mainly focus on three different categories from small to large: Kazhdan-Lusztig category, category $\mathcal{O}$, and the category of weight modules. I will also discuss their relations to the representations of quantum $SL(2)$ groups.

Zhu Algebras of Permutation Orbifold Vertex Operator Algebras

余铌娜 (厦门大学)

In this talk, we will discuss the Zhu algebras of permutation orbifold vertex operator algebras and related topics.

同调维数与模型结构

章　璞 (上海交通大学)

模型结构在数学的众多领域有广泛而深刻的应用。在这个报告中我们将说明，各种同调维数 (投射，内射，平坦，Gorenstein 投射，Gorenstein 内射，Gorenstein 平坦，PGF) 有限的模，诱导出一系列新的模型结构。

例如，对 Artin 代数 R，A. Beligiannis 和 I. Reiten 证明了 $(\mathcal{GP}, \mathcal{GP}^{\perp})$ 是 R-Mod 中的完备和遗传余挠对，其中 $\mathcal{GP}$ 是 Gorenstein 投射 R-模作成的类。2020 年 J. Šaroch 和 J. Štovíček 引入 PGF 模。他们证明了 $(\mathcal{PGF}, \mathcal{PGF}^{\perp})$ 是 R-Mod 中的完备和遗传余挠对，其中 $\mathcal{PGF}$ 是 PGF R-模作成的类。

我们指出，对任意环 R 任意非负整数 n，$(\mathcal{PGF}_n, \mathcal{P}_n^{\perp} \cap \mathcal{PGF}^{\perp})$ 都是 R-Mod 中的完备和遗传余挠对, 其中 $\mathcal{PGF}_n$ 是 PGF 维数 $\leq n$ 的模作成的类。这诱导出遗传模型结构 (Hovey 三元组) $(\mathcal{PGF}_n, \mathcal{P}_n^{\perp}, \mathcal{PGF}^{\perp})$. 它们都具有相同的同伦范畴 $\mathcal{PGF}/\mathcal{P}$.

各种同调维数与模型结构的关系如下图所示.

Table 1: Complete and hereditary cotorsion pairs and the induced Hovey triples in R-Mod

	$n=0$	n	$<\infty$
$\mathcal{P}$	$(\mathcal{P},\ R\text{-Mod})$	$(\mathcal{P}_n,\ \mathcal{P}_n^{\perp})$ Enochs, Jenda	If Fpd $<\infty$, then $(\mathcal{P}^{<\infty},\ (\mathcal{P}^{<\infty})^{\perp})$
$\mathcal{FL}$ $=\mathcal{F}_0$	$(\mathcal{FL},\ \mathcal{EC})$ Enochs, Jenda	$(\mathcal{F}_n,\ \mathcal{F}_n^{\perp})$ Mao, Ding	If Fpd $<\infty$, then $(\mathcal{F}^{<\infty},\ (\mathcal{F}^{<\infty})^{\perp})$
$\mathcal{PGF}$	$(\mathcal{PGF},\ \mathcal{PGF}^{\perp})$ $(\mathcal{PGF},\ R\text{-Mod},\ \mathcal{PGF}^{\perp})$ Šaroch, Šťovíček	$(\mathcal{PGF}_n,\ \mathcal{P}_n^{\perp}\cap\mathcal{PGF}^{\perp})$ $(\mathcal{PGF}_n,\ \mathcal{P}_n^{\perp},\ \mathcal{PGF}^{\perp})$	If Fpd $<\infty$, then $(\mathcal{PGF}^{<\infty},\ (\mathcal{P}^{<\infty})^{\perp}\cap\mathcal{PGF}^{\perp})$ $(\mathcal{PGF}^{<\infty},\ (\mathcal{P}^{<\infty})^{\perp},\ \mathcal{PGF}^{\perp})$
$\mathcal{GP}$	For Artin algebras $(\mathcal{GP},\ \mathcal{GP}^{\perp})$ $(\mathcal{GP},\ R\text{-Mod},\ \mathcal{GP}^{\perp})$ Beligiannis, Reiten	For Artin algebras $(\mathcal{GP}_n,\ \mathcal{P}_n^{\perp}\cap\mathcal{GP}^{\perp})$ $(\mathcal{GP}_n,\ \mathcal{P}_n^{\perp},\ \mathcal{GP}^{\perp})$	For Artin algebras If Fpd $<\infty$, then $(\mathcal{GP}^{<\infty},\ (\mathcal{P}^{<\infty})^{\perp}\cap\mathcal{GP}^{\perp})$ $(\mathcal{GP}^{<\infty},\ (\mathcal{P}^{<\infty})^{\perp},\ \mathcal{GP}^{\perp})$
$\mathcal{GF}$	$(\mathcal{GF},\ \mathcal{EC}\cap\mathcal{PGF}^{\perp})$ $(\mathcal{GF},\ \mathcal{EC},\ \mathcal{PGF}^{\perp})$ Šaroch, Šťovíček	$(\mathcal{GF}_n,\ \mathcal{F}_n^{\perp}\cap\mathcal{PGF}^{\perp})$ $(\mathcal{GF}_n,\ \mathcal{F}_n^{\perp},\ \mathcal{PGF}^{\perp})$ Maaouy	If Fpd $<\infty$, then $(\mathcal{GF}^{<\infty},\ (\mathcal{F}^{<\infty})^{\perp}\cap\mathcal{PGF}^{\perp})$ $(\mathcal{GF}^{<\infty},\ (\mathcal{F}^{<\infty})^{\perp},\ \mathcal{PGF}^{\perp})$

另一方面，正合范畴是 Abel 范畴的推广。 Abel 范畴的对扩张和直和项封闭的满子范畴，都是弱幂等完备的正合范畴，却未必是 Abel 范畴。

对任意环 R 和任意整数 $n\geq 0$, 我们在 6 种弱幂等完备正合范畴 $\mathcal{PGF}_n$, $\mathcal{GP}_n$, $\mathcal{GF}_n$, $\mathcal{PGF}^{<\infty}$, $\mathcal{GP}^{<\infty}$ 和 $\mathcal{GF}^{<\infty}$, 都得到相应的余挠对和正合模型结构。这基于如下观察：若 $\mathcal{B}$ 是 Abel 范畴 $\mathcal{A}$ 的对扩张和满态射的核封闭的全子范畴, 则 $\mathcal{A}$ 中完备余挠对 $(\mathcal{X},\ \mathcal{Y})$，其中 $\mathcal{X}\subseteq\mathcal{B}$, 诱导出正合范畴 $\mathcal{B}$ 的完备余挠对 $(\mathcal{X},\ \mathcal{Y}\cap\mathcal{B})$；若 $(\mathcal{X},\ \mathcal{Y})$ 遗传, 则 $(\mathcal{X},\ \mathcal{Y}\cap\mathcal{B})$ 也遗传.

例如, 将这一观察应用于 R-Mod 中的完备遗传余挠对 $(\mathcal{PGF}_m,\ \mathcal{P}_m^{\perp}\cap\mathcal{PGF}^{\perp})$, 我们得到 $\mathcal{PGF}^{<\infty}$ 中的完备遗传余挠对: $(\mathcal{PGF}_m,\ \mathcal{P}_m^{\perp}\cap\mathcal{P}^{<\infty})$, 和遗传模型结构 $(\mathcal{PGF}_m,\ \mathcal{P}_m^{\perp}\cap\mathcal{PGF}^{<\infty},\ \mathcal{P}^{<\infty})$. 这推广了 G. Dalezios 和 I. Emmanouil 在 $m=0$ 时的结果。

更有趣, 对任意整数 $0\leq m\leq n$, $(\mathcal{GP}_m,\ \mathcal{P}_m^{\perp}\cap\mathcal{P}_n)$ 是 $\mathcal{GP}_n$ 中的完备遗传余挠对, $(\mathcal{GP}_m,\ \mathcal{P}_m^{\perp}\cap\mathcal{GP}_n,\ \mathcal{P}_n)$ 是 $\mathcal{GP}_n$ 中的遗传模型结构 (Hovey 三元组). 这里 R 是任意环, 不必是 Artin 代数。

6 种由同调维数确定的弱幂等完备正合范畴中的模型结构如下图所示.

Table 2: Complete and hereditary cotorsion pairs and the induced Hovey triples in exact categories

	$n=0$	n $0\le m\le n$	$<\infty$ m
$\mathcal{P}$	In $\mathcal{P}$ $(\mathcal{P},\ \mathcal{P})$	In $\mathcal{P}_n$ $(\mathcal{P}_m,\ \mathcal{P}_m^{\perp}\cap\mathcal{P}_n)$	In $\mathcal{P}^{<\infty}$ $(\mathcal{P}_m,\ \mathcal{P}_m^{\perp}\cap\mathcal{P}^{<\infty})$
$\mathcal{FL}=\mathcal{F}_0$	In $\mathcal{FL}$ $(\mathcal{FL},\ \mathcal{EC}\cap\mathcal{FL})$	In $\mathcal{F}_n$ $(\mathcal{P}_m,\ \mathcal{P}_m^{\perp}\cap\mathcal{F}_n)$ $(\mathcal{F}_m,\ \mathcal{F}_m^{\perp}\cap\mathcal{F}_n)$	In $\mathcal{F}^{<\infty}$ $(\mathcal{P}_m,\ \mathcal{P}_m^{\perp}\cap\mathcal{F}^{<\infty})$ $(\mathcal{F}_m,\ \mathcal{F}_m^{\perp}\cap\mathcal{F}^{<\infty})$
$\mathcal{PGF}$	In $\mathcal{PGF}$ $(\mathcal{PGF},\ \mathcal{P})$ $(\mathcal{PGF},\ \mathcal{PGF},\ \mathcal{P})$	In $\mathcal{PGF}_n$ $(\mathcal{P}_m,\ \mathcal{P}_m^{\perp}\cap\mathcal{PGF}_n)$ $(\mathcal{PGF}_m,\ \mathcal{P}_m^{\perp}\cap\mathcal{P}_n)$ $(\mathcal{PGF}_m,\ \mathcal{P}_m^{\perp}\cap\mathcal{PGF}_n,\ \mathcal{P}_n)$	In $\mathcal{PGF}^{<\infty}$ $(\mathcal{P}_m,\ \mathcal{P}_m^{\perp}\cap\mathcal{PGF}^{<\infty})$ $(\mathcal{PGF}_m,\ \mathcal{P}_m^{\perp}\cap\mathcal{P}^{<\infty})$ $(\mathcal{PGF}_m,\ \mathcal{P}_m^{\perp}\cap\mathcal{PGF}^{<\infty},\ \mathcal{P}^{<\infty})$
$\mathcal{GP}$	In $\mathcal{GP}$ $(\mathcal{PGF},\ \mathcal{PGF}^{\perp}\cap\mathcal{GP})$ $(\mathcal{GP},\ \mathcal{P})$ $(\mathcal{PGF},\ \mathcal{GP},\ \mathcal{PGF}^{\perp}\cap\mathcal{GP})$ $(\mathcal{GP},\ \mathcal{GP},\ \mathcal{P})$	In $\mathcal{GP}_n$ $(\mathcal{P}_m,\ \mathcal{P}_m^{\perp}\cap\mathcal{GP}_n)$ $(\mathcal{PGF}_m,\ \mathcal{P}_m^{\perp}\cap\mathcal{PGF}^{\perp}\cap\mathcal{GP}_n)$ $(\mathcal{GP}_m,\ \mathcal{P}_m^{\perp}\cap\mathcal{P}_n)$ $(\mathcal{PGF}_m,\ \mathcal{P}_m^{\perp}\cap\mathcal{GP}_n,\ \mathcal{PGF}^{\perp}\cap\mathcal{GP}_n)$ $(\mathcal{GP}_m,\ \mathcal{P}_m^{\perp}\cap\mathcal{GP}_n,\ \mathcal{P}_n)$	In $\mathcal{GP}^{<\infty}$ $(\mathcal{P}_m,\ \mathcal{P}_m^{\perp}\cap\mathcal{GP}^{<\infty})$ $(\mathcal{PGF}_m,\ \mathcal{P}_m^{\perp}\cap\mathcal{PGF}^{\perp}\cap\mathcal{GP}^{<\infty})$ $(\mathcal{GP}_m,\ \mathcal{P}_m^{\perp}\cap\mathcal{P}^{<\infty})$ $(\mathcal{PGF}_m,\ \mathcal{P}_m^{\perp}\cap\mathcal{GP}^{<\infty},\ \mathcal{PGF}^{\perp}\cap\mathcal{GP}^{<\infty})$ $(\mathcal{GP}_m,\ \mathcal{P}_m^{\perp}\cap\mathcal{GP}^{<\infty},\ \mathcal{P}^{<\infty})$
$\mathcal{GF}$	In $\mathcal{GF}$ $(\mathcal{FL},\ \mathcal{EC}\cap\mathcal{GF})$ $(\mathcal{GF},\ \mathcal{FL}\cap\mathcal{EC})$ $(\mathcal{GF},\ \mathcal{EC}\cap\mathcal{GF},\ \mathcal{FL})$ $(\mathcal{PGF},\ \mathcal{FL})$ $(\mathcal{PGF},\ \mathcal{GF},\ \mathcal{FL})$ Dalezios, Emmanouil	In $\mathcal{GF}_n$ $(\mathcal{P}_m,\ \mathcal{P}_m^{\perp}\cap\mathcal{GF}_n)$ $(\mathcal{F}_m,\ \mathcal{F}_m^{\perp}\cap\mathcal{GF}_n)$ $(\mathcal{PGF}_m,\ \mathcal{F}_n\cap\mathcal{P}_m^{\perp})$ $(\mathcal{GF}_m,\ \mathcal{F}_n\cap\mathcal{F}_m^{\perp})$ $(\mathcal{PGF}_m,\ \mathcal{P}_m^{\perp}\cap\mathcal{GF}_n,\ \mathcal{F}_n)$ $(\mathcal{GF}_m,\ \mathcal{F}_m^{\perp}\cap\mathcal{GF}_n,\ \mathcal{F}_n)$	In $\mathcal{GF}^{<\infty}$ $(\mathcal{P}_m,\ \mathcal{P}_m^{\perp}\cap\mathcal{GF}^{<\infty})$ $(\mathcal{F}_m,\ \mathcal{F}_m^{\perp}\cap\mathcal{GF}^{<\infty})$ $(\mathcal{PGF}_m,\ \mathcal{F}^{<\infty}\cap\mathcal{P}_m^{\perp})$ $(\mathcal{GF}_m,\ \mathcal{F}^{<\infty}\cap\mathcal{F}_m^{\perp})$ $(\mathcal{PGF}_m,\ \mathcal{P}_m^{\perp}\cap\mathcal{GF}^{<\infty},\ \mathcal{F}^{<\infty})$ $(\mathcal{GF}_m,\ \mathcal{F}_m^{\perp}\cap\mathcal{GF}^{<\infty},\ \mathcal{F}^{<\infty})$

Reduction for flag-transitive symmetric 2-designs with $\gcd(k,\lambda)=p$

周胜林 (华南理工大学)

Let $\mathcal{D}=(\mathcal{P},\mathcal{B})$ be a 2-(v,k,λ) design. Each point of $\mathcal{D}$ is contained in exactly r blocks. In this talk, we study flag-transitive symmetric 2-(v,k,λ) designs $\mathcal{D}$ with $\gcd(r,\lambda)=p$, where p is a prime. It is proved that if $G\le Aut(\mathcal{D})$ is flag-transitive and point-primitive, then $Soc(G)$ is an elementary abelian group or a non-abelian simple

group. This is a joint work with Dr. Ziwei Lu.