天元數學國際交流中心
Tianyuan Mathematics Research Center

Seminar on the Mathematical Theory of Compressible Fluids

Conveners: Feimin Huang, Yong Wang

Conference Handbook

Kunming · Yunnan

July 13–19, 2025

Seminar Schedule

Sunday, July 13, 2025	**Registration and Check-in**
Saturday, July 19, 2025	**Departure**

Monday, July 14, 2025

8:45 am - 9:00 am	**OPENING and GROUP PHOTO**	
9:00 am - 10:00 am	**Speaker: Yongqian Zhang**	**Chair: Feimin Huang**
10:00 am - 10:30 am	**TEA BREAK**	
10:30 am - 11:30 am	**Speaker: Hairong Yuan**	**Chair: Yongqian Zhang**
11:30 am - 15:00 pm	**LUNCH BREAK**	
15:00 pm - 16:00 pm	**Speaker: Aifang Qu**	**Chair: Hairong Yuan**
16:00 pm - 16:30 pm	**TEA BREAK**	
16:30 pm - 17:30 pm	**Speaker: Rui Li**	**Chair: Aifang Qu**
17:30 pm	**DINNER BREAK**	

Tuesday, July 15, 2025

9:00 am - 10:00 am	**Speaker: Beixiang Fang**	**Chair: Yong Wang**
10:00 am - 10:30 am	**TEA BREAK**	
10:30 am - 11:30 am	**Speaker: Wei Xiang**	**Chair: Beixiang Fang**
11:30 am - 15:00 pm	**LUNCH BREAK**	
15:00 pm - 16:00 pm	**Speaker: Tianyi Wang**	**Chair: Wei Xiang**
16:00 pm - 16:30 pm	**TEA BREAK**	
16:30 pm - 17:30 pm	**Speaker: Jing Ouyang**	**Chair: Hao Zheng**
17:30 pm	**DINNER BREAK**	

Wednesday, July 16, 2025

9:00 am - 10:00 am	Speaker: Geng Lai	Chair: Tianyi Wang
10:00 am - 10:30 am	TEA BREAK	
10:30 am - 11:30 am	Speaker: Tao Wang	Chair: Geng Lai
11:30 am - 15:00 pm	LUNCH BREAK	
15:00 pm - 17:30 pm	FREE DISCUSSION	
17:30 pm	DINNER BREAK	

Thursday, July 17, 2025

9:00 am - 10:00 am	Speaker: Jianfeng Cheng	Chair: Jie Kuang
10:00 am - 10:30 am	TEA BREAK	
10:30 am - 11:30 am	Speaker: Hao Zheng	Chair: Feng Xiao
11:30 am - 15:00 pm	LUNCH BREAK	
15:00 pm - 17:30 pm	FREE DISCUSSION	
17:30 pm	DINNER BREAK	

Friday, July 18, 2025

9:00 am - 11:30 am	FREE DISCUSSION
11:30 am - 15:00 pm	LUNCH BREAK
15:00 pm - 17:30 pm	FREE DISCUSSION
17:30 pm	DINNER BREAK

Report Information

Jianfeng Cheng, Sichuan University, China

Title: The incompressible jet flow with vorticity and gravity

Abstract: This talk is concerned with the well-posedness theory and geometric property of steady incompressible jet flow with vorticity and gravity. The main results show that for given incoming mass flux and atmospheric pressure at outlet there exists a unique smooth incompressible jet flow issuing from a semi-infinitely long nozzle. Moreover, we obtain the single intersection property and monotonicity of the free boundary.

Beixiang Fang, Shanghai Jiao Tong University, China

Title: Solutions for steady 3-D axisymmetric full Euler flows with swirl velocity in a finite cylindrical nozzle

Abstract: This talk is concerned with the existence and location of three-dimensional axisymmetric transonic shocks with large swirl velocity for solutions of the steady compressible full Euler system in a cylindrical nozzle with prescribed receiver pressure. Special non-trivial shock solutions with large vorticity are first constructed by considering arbitrarily given non-zero swirl functions. Then the existence and locations of the transonic shock solutions to the full Euler equations are achieved under small perturbations on the special shock solutions with appropriate boundary conditions on the entrance of the nozzle and the receiver pressure at the exit. It turns out that the non-zero swirl velocity, which brings new challenging difficulties in the analysis, plays an essential and fundamental role in determining the location of the shock front. This talk is based on a joint work with Xin Gao, Wei Xiang, and Qin Zhao.

Geng Lai, Shanghai University, China

Title: Wave interactions in 2D supersonic flows of BZT fluids

Abstract: In this talk, I will introduce the interaction of fan-shock-fan composite waves in 2D pseudo-steady supersonic flows of Bethe-Zel'dovich-Thompson (BZT) fluids. In contrast to ideal polytropic gases, physically admissible shocks of BZT fluids may be transonic, post-sonic, pre-sonic, or double-sonic in the sense of the flow velocity relative to the shock front. The shocks of the fan-shock-fan composite waves are double-sonic. However, when the shocks enter into the fan-shock-fan wave interaction region, the type of the shocks is actually a priori unknown. Hence, the formulation of the boundary conditions on the shocks in the interaction region is also a priori unknown. By calculating the curvatures of the shocks and using the Liu's extended entropy condition, we prove that the shocks in the interaction region must be post-sonic. We prove that the shocks are envelopes of one out of the two families of wave characteristics of the flow downstream of them, and not characteristics. This leads to a fact that the flow downstream of the shocks is not C^1 smooth up to the shock boundaries. We will introduce the hodograph transformation method and the characteristic decomposition method to overcome the difficulty

caused by the singularity. Some other wave interactions of BZT fluids arising in 2D Riemann problems and 2D steady supersonic nozzle flows will also be introduced.

Rui Li, Academy of Mathematics and Systems Science, CAS, China
Title: Nonlinear stability and optimal decay rate of the planar entropy wave
Abstract: This talk introduces the stability and optimal decay rate of planar entropy waves for the Navier-Stokes equations and the Landau equation. Since Liu and Xin (1997) proposed the left/right structural conditions in their study of the metastability of contact discontinuities, subsequent research on large-time stability has often relied on these conditions. For the Navier-Stokes equations in Eulerian coordinates, we present a transformation that converts systems satisfying the structural condition only on one side into new systems satisfying it bilaterally. For the Landau equation, we address the challenge of weak collisional dissipation by employing coupled diffusion waves and a time-microscopic-velocity transformation technique. This enables us to develop an anti-derivative method and establish the nonlinear stability and optimal decay rate of multidimensional entropy waves for the Landau equation. This is a joint work with Prof. Renjun Duan, Prof. Feimin Huang, and Dr. Lingda Xu.

Jing Ouyang, Institute of Applied Physics and Computational Mathematics, Beijing, China
Title: The Hydrodynamic Limit Problem for the Boltzmann Equation
Abstract: The GHW-2021-ARMA article systematically established the Hilbert expansion from the Boltzmann equation to the compressible Euler equations in a half-space with specular reflection boundary conditions. It derived the interior layer, viscous layer, and Knudsen layer, and rigorously justified the corresponding hydrodynamic limit. That work primarily addressed the hard sphere collision model. Through meticulous analysis, we extend these results to both hard and soft potentials. Our methodology remains equally applicable to the hard sphere case. Furthermore, we consider curved surfaces with curvature — specifically, the archetypal two-dimensional disk. During this investigation, we revealed novel phenomena within the Knudsen layer. By developing an innovative framework, we establish the existence, uniqueness, and decay properties of the Knudsen layer. This enables us to complete the Hilbert expansion and rigorously validate the hydrodynamic limit from the Boltzmann equation to the compressible Euler equations on a two-dimensional disk.

Aifang Qu, Shanghai Normal University, China
Title: Singular Riemann problems and their applications
Abstract: In this talk, we focus on singular Riemann problems for the compressible Euler equations. We will discuss the proper integral form of the fundamental equations for complete and general description of ideal fluid flow with all its discontinuities including concentration. We will also introduce some interesting applications of these problems. This talk is based on joint work with Professor Hairong Yuan (ECNU).

Tao Wang, Wuhan University, China

Title: Vacuum free boundary problems in ideal compressible MHD

Abstract: We present the joint works with Paolo Secchi and Yuri Trakhinin on local well-posedness of vacuum free boundary problems in non-relativistic and relativistic ideal compressible magnetohydrodynamics (MHD).

Tianyi Wang, Wuhan University of Technology, China

Title: On vanishing pressure limit of continuous solutions to the isentropic Euler equations and related model

Abstract: The vanishing pressure limit of continuous solutions isentropic Euler equations is analyzed, which is formulated as small parameter goes to 0. Due to the characteristics being degenerated in the limiting process, the resonance may cause the mass concentration. It is shown that in the pressure vanishing process, for the isentropic Euler equations, the continuous solutions with compressive initial data converge to the mass concentration solution of pressureless Euler equations, and with rarefaction initial data converge to the continuous solutions globally. Then, I will mention the recent progress on the hypersonic limit and Aw-Rascle traffic flow model.

Wei Xiang, City University of Hong Kong, China

Title: Regular shock reflection problem

Abstract: We will talk about our recent results on the regular reflection solutions for the potential flow equation and Euler equations in a natural class of self-similar solutions.

Hairong Yuan, East China Normal University, China

Title: Radon measure-valued solutions of compressible Euler equations and concentration boundary layers in unsteady inviscid flows passing solid obstacles

Abstract: For time-dependent compressible Euler flows passing around a fixed solid body in three-dimensional space, there may exist an infinitesimally thin layer of concentrated mass, momentum and energy, wherein all particles impacting the body move along the body's windward boundary surface. By proposing a concept of Radon measure-valued solutions for initial-boundary-value problems of the unsteady compressible Euler equations, which captures both the large-scale three-dimensional distributions of the surrounding flows and the small-scale motions of particles on the two-dimensional boundary surfaces, we derive the governing partial differential equations for the concentration boundary layer --- an unsteady (pressureless) compressible Euler system defined on the boundary surface with appropriate source terms. This down-scaling approach can be further generalized to incorporate skin-frictions and phase-transitions within the concentration boundary layer. It constitutes a novel methodology for addressing the complex fluid-solid-heat coupling problems encountered in fluid dynamics. Illustrative examples are presented to demonstrate the applicability of the proposed method to several specific problems, including the Newtonian-Busemann pressure laws of hypersonic

aerodynamics.

Yongqian Zhang, Fudan University, China
Title: Some direct and inverse problems for steady supersonic flows
Abstract: We will talk about some direct and inverse problems for steady supersonic flow past a wedge, or a bending walls, and present recent progress on the existence and stability results for the inverse problem of determining the shape of the wedge or the bending wall with the desired pressure distribution in the steady supersonic flow.

Hao Zheng, Academy of Mathematics and Systems Science, CAS, China
Title: Analysis on the initial layer of time-relaxation limit of the quantum hydrodynamic equation
Abstract: In this talk we will present some recent result on the structure analysis on the initial layer phenomena of the time-relaxation limit of the quantum hydrodynamic (QHD) equation for semi-conductor devices. As the counterpart of the time-relaxation limit problem in classical fluid models, the mismatch of initial data of the fluid model and its singular limit leads to the general existence of a thin layer at initial time, where the momentum density changes drastically, unless one assumes the initial data is well-prepared. In our previous work (Antonelli, Marcati, Z. arXiv.2412.16800), we have rigorously established the result of the time-relaxation limit of the QHD for ill-prepared initial data, and an explicit convergence rate is obtained for the first time. Recently, we have produced further analysis on the structure of the initial layer by using the method of parameter expansion, and proved the boundedness of the remainders. This result implies the optimality of our previous convergence rate.