

# Conference on Relative Langlands Program August 3--August 16, 2025 Tianyuan Mathematics Research Center







### **Organizing Committee**

Ning Li, Nankai University

Dongwen Liu, Zhejiang University

Jia-Jun Ma, Xiamen University

Binyong Sun, IASM, Zhejiang University

Lei Zhang, National University of Singapore

## Week 1

Date Time	Aug 4 Monday	Aug 5 Tuesday	Aug 6 Wednesday	Aug 7 Thursday	Aug 8 Friday
9:00-10:00	BZSV I	BZSV II	BZSV III	BZSV IV	Li Cai
10:00-10:30			Tea Break		
10:30-11:30	Chengbo Zhu	Chong Zhang	Jing-Song Huang	Bin Xu (Tsinghua)	Yangyu Fan
11:30-13:30	Lunch and Break				
13:30-15:30	Group Discussion (Level 1 Discussion rooms)				
13.30-15.30	Dongwen Liu, Weixiao Lu, Jia-Jun Ma, Yangyu Fan, Caihua Luo				
15:30-16:30	Tea Break and Discussion				
16:30-17:00	Meeting				

## Week 2

Date Time	Aug 11 Monday	Aug 12 Tuesday	Aug 13 Wednesday	Aug 14 Thursday	Aug 15 Friday
9:00-10:00	Hongfeng Zhang (2:30pm)	Colin Loh	Kei Yuen Chan	Bin Xu (Sichuan)	Jia-Jun Ma
10:00-10:30	Tea Break				
10:30-11:30	Qiutong Wang (4:00pm)	Guodong Xi	Rui Chen	Hengfei Lu	Zhicheng Wang
11:30-13:30	Lunch and Break				
13:30-15:30	Group Discussion (Level 1 Discussion rooms)				
13.30-15.30	Dongwen Liu, Weixiao Lu, Jia-Jun Ma, Yangyu Fan, Caihua Luo				
15:30-16:30	Tea Break and Discussion				
16:30-17:00	Meeting				

## August 4, Monday

Time	Speaker	Title	
9:00-10:00	Lei Zhang	Overview of Relative Langlands Duality	
10:00-10:30		Tea Break	
10:30-11:30	Chengbo Zhu	The first and second fundamental theorems of invariant theory: the distributional version	
11:30-13:30	Lunch and Break		
13:30-15:30	Group Discussion (Level 1 Discussion room)		
Group chair	Dongwen Liu, Weixiao Lu, Jia-Jun Ma, Yangyu Fan, Caihua Luo		
15:30-16:30	Tea Break and Discussion		
16:30-17:00	Meeting		

## August 5, Tuesday

Time	Speaker	Title	
9:00-10:00	Lei Zhang	Hyperspherical Hamiltonian spaces	
10:00-10:30		Tea Break	
10:30-11:30	Chong Zhang	Local theta correspondence and local periods	
11:30-13:30	Lunch and Break		
13:30-15:30	Group Discussion (Level 1 Discussion room)		
Group chair	Dongwen Liu, Weixiao Lu, Jia-Jun Ma, Yangyu Fan, Caihua Luo		
15:30-16:30	Tea Break and Discussion		
16:30-17:00	Meeting		

## August 6, Wednesday

Time	Speaker	Title	
9:00-10:00	Lei Zhang	Distinguished polarized hyperspherical vareiteis	
10:00-10:30	Tea Break		
10:30-11:30	Jing-Song Huang Principal Elements and Fourier Operators		
11:30-13:30	Lunch and Break		
13:30-17:30	Free Discussion		

## **August 7, Thursday**

Time	Speaker	Title	
9:00-10:00	Lei Zhang	Numerical global and local conjecture	
10:00-10:30		Tea Break	
10:30-11:30	Bin Xu (Tsinghua)	Nontempered GGP in the framework of relative Langlands duality	
11:30-13:30	Lunch and Break		
13:30-15:30	Group Discussion (Level 1 Discussion room)		
Group chair	Dongwen Liu, Weixiao Lu, Jia-Jun Ma, Yangyu Fan, Caihua Luo		
15:30-16:30	Tea Break and Discussion		
16:30-17:00	Meeting		

## August 8, Friday

Time	Speaker	Title	
9:00-10:00	Li Cai	The relative trace formula approach: the Gan- Gross-Prasad case	
10:00-10:30		Tea Break	
10:30-11:30	Yangyu Fan	Test vectors for Rankin pairs	
11:30-13:30	Lunch and Break		
13:30-15:30	Group Discussion (Level 1 Discussion room)		
Group chair	Dongwen Liu, Weixiao Lu, Jia-Jun Ma, Yangyu Fan, Caihua Luo		
15:30-16:30	Tea Break and Discussion		
16:30-17:00	Meeting		

## August 11, Monday

Time	Speaker	Title
9:00-11:30	Free Discussion	
11:30-14:30	Lunch and Break (Level 1 Discussion room)	
14:30-15:30	Hongfeng Zhang	TBA
15:30-16:00	Tea Break	
16:00-17:00	Qiutong Wang	Counting irreducible representations of general linear groups and unitary groups

## August 12, Tuesday

Time	Speaker	Title	
9:00-10:00	Colin Loh	Period integrals associated to dual of strongly tempered hyperspherical variety	
10:00-10:30		Tea Break	
10:30-11:30	Guodong Xi	Regularization of period integrals and examples of BZSV conjectures	
11:30-13:30	Lunch and Break		
13:30-15:30	Group Discussion (Level 1 Discussion room)		
Group Chair	Dongwen Liu, Weixiao Lu, Jia-Jun Ma, Yangyu Fan, Caihua Luo		
15:30-16:30	Tea Break and Discussion		
16:30-17:00	Meeting		

## August 13, Wednesday

Time	Speaker	Title	
9:00-10:00	Kei Yuen Chan	The product functor method in the non-tempered branching laws	
10:00-10:30	Tea Break		
10:30-11:30	Rui Chen On non-tempered GGP problem		
11:30-13:30	Lunch and Break		
13:30-17:30	Free Discussion		

## **August 14, Thursday**

Time	Speaker	Title	
9:00-10:00	Bin Xu (Sichuan)	The local descent construction for classical groups	
10:00-10:30		Tea Break	
10:30-11:30	Hengfei Lu	The intertwining period	
11:30-13:30	Lunch and Break		
13:30-15:30	Group Discussion (Level 1 Discussion room)		
Group Chair	Dongwen Liu, Weixiao Lu, Jia-Jun Ma, Yangyu Fan, Caihua Luo		
15:30-16:30	Tea Break and Discussion		
16:30-17:00	Meeting		

## August 15, Friday

Time	Speaker	Title	
9:00-10:00	Jia-Jun Ma	Hecke Algebra Approach to Local Theta Correspondence	
10:00-10:30		Tea Break	
10:30-11:30	Zhicheng Wang	Branching Problems over Finite Fields	
11:30-13:30	Lunch and Break		
13:30-15:30	Group Discussion (Level 1 Discussion room)		
Group Chair	Dongwen Liu, Weixiao Lu, Jia-Jun Ma, Yangyu Fan, Caihua Luo		
15:30-16:30	Tea Break and Discussion		
16:30-17:00	Meeting		

## Title and Abstract Week 1

#### Li Cai (Capital Normal University)

Title: The relative trace formula approach: the Gan-Gross-Prasad case

**Abstract:** The relative trace formula is an approach, introduced by Jacquet, to study period integrals. In this talk, we shall review the proof of the GGP conjecture for U(n) times U(n+1) via the RTF approach. Especially, we shall sketch the proof of some key ingredients: the fundamental lemma, the existence of smooth matching and the spectral expansion.

#### Yangyu Fan (Beijing Institute of Technology)

**Title:** Test vectors for Rankin pairs

**Abstract:** In this talk, we shall survey some recent results on test vectors concerning special value formulae for Rankin pairs.

#### Jing-Song Huang (The Chinese University of Hong Kong, Shenzhen)

**Title:** Principal Elements and Fourier Operators

**Abstract:** Let g be a complex simple Lie algebra and G the adjoint group of g. The principal element of G corresponds to the Coxeter element and the Fourier element corresponds to the longest element of the Weyl group. We define the Fourier operators for a split real form G(R) and show that they act on unitary representations of G(R) in a way similar to the classical Fourier transforms on the oscillator representation of the symplectic group. As a consequence, it reveals the spectral decomposition of the Fourier transforms. This framework is extended to quantum Fourier transforms in superspace analysis.

#### Bin Xu (Tsinghua University)

**Title:** Nontempered GGP in the framework of relative Langlands duality

**Abstract:** This is an expository talk aimed at reinterpreting the nontempered GGP in the framework of relative Langlands duality proposed by Ben-Zvi, Sakellaridis and Venkatesh.

#### **Chong Zhang (Nanjing University)**

Title: Local theta correspondence and local periods

**Abstract:** Theta correspondence can be used to study the relation between certain periods of the representations of the reductive dual pairs. I will report some recent advances on this topic.

#### Lei Zhang (National University of Singapore)

**Title:** Introduction to Relative Langlands Duality I–V

**Abstract:** In the first week, we will have a 4-hour introduction to Relative Langlands Duality. We will cover the definition of Hyperspherical varieties, Distinguished polarized hyperspherical varieties and their dual, and the numerical conjecture in local and global conjectures. The goal of the introductory course is to connect the other conference topics together and stimulate the discussions from different points of view.

#### **Chen-Bo Zhu (National University of Singapore)**

**Title:** The first and second fundamental theorems of invariant theory: the distributional version

**Abstract:** I will explain some old results of Kudla-Rallis as well as Lee and myself, on the structure of degenerate principal series, which I view as the transcendental versions of the first and second fundamental theorems of invariant theory. I will then explain some (recent and not so recent) applications of these results in extracting qualitative and quantitative properties of theta liftings such as nonvanishing and associated/wavefront cycles.

#### Week 2

#### **Kei Yuen Chan (The University of Hong Kong)**

**Title:** The product functor method in the non-tempered branching laws

**Abstract:** A key ingredient in a proof of the non-tempered branching law for general linear groups is a functorial property of parabolic induction. In this talk, I will explain this method and some generalizations on that. Some new results are joint with Saad Qadri.

#### Rui Chen (Zhejiang University)

Title: On non-tempered GGP problem

**Abstract:** A much studied branching problem in representation theory of reductive groups is the so called Gan--Gross--Prasad (GGP) problem. The original GGP problem concerns tempered representations, and recently it has been extended to Arthur type representations. In this talk we will review the current status of the non-tempered GGP problem, and present some recent progress on this problem and a twisted variant of it.

#### Colin Loh(National University of Singapore)

**Title:** Period integrals associated to dual of strongly tempered hyperspherical variety

**Abstract:** Recent work of Mao, Wan and Zhang have provided a complete list of strongly tempered hyperspherical varieties and they proposed some new period integrals. In this talk, I will present some new period integrals associated to dual of strongly tempered distinguished polarised hyperspherical varieties and discuss the L-functions these integrals represent, as examples of the Relative Langlands Duality.

#### Hengfei Lu (Beihang University)

Title: The intertwining period

**Abstract:** Let G/H be a symmetric variety over a p-adic local field F. Suppose that Pi is a normalized induced representation of G. We will discuss the relations between the meromorphic continuation of the functionals coming from the open orbits and the multiplicities.

#### Jia-Jun Ma (Xiamen University)

Title: Hecke Algebra Approach to Local Theta Correspondence

Abstract: Theta correspondence provides a correspondence between representations of certain pairs of reductive groups. It serves as a fundamental tool in the representation theory of reductive groups and has numerous applications to the Langlands program as it is a good approximation of certain Langlands functoriality. A central open question in this field concerns the structure of the big theta lift (i.e., the Howe maximal quotient) of an irreducible representation. The category of representations of reductive groups over finite or p-adic fields decomposes in terms of cuspidal support. Each component is equivalent to the module category of a certain Hecke algebra. Since theta correspondence respects this decomposition, computing the big theta lift reduces to determining a correspondence between Hecke algebra modules. In this talk, I will discuss our recent results in this direction: (1) the finite field case (joint work with Qiu and Zou), (2) the geometrization of the finite field picture and its relation with Springer theory (joint work with Qiu, Yun, and Zou), and (3) the picture in the p-adic field case (joint work with Loke, Stevens, and Trias). We hope this approach can offer new insights and eventually lead to resolving this longstanding question.

#### **Guodong Xi (University of Minnesota)**

**Title:** Regularization of period integrals and examples of BZSV conjectures

**Abstract:** A central problem in the relative Langlands program, recently developed by Ben-Zvi, Sakellaridis, and Venkatesh, is to establish the equality between certain period integrals and L-functions. This conjecture was further refined by Mao, Wan, and Zhang. In this talk, I will present numerical evidence supporting specific cases of the BZSV duality.

#### Bin Xu (Sichuan University)

**Title:** The local descent construction for classical groups

**Abstract:** In this talk, we will introduce the local descent construction from representations of GL(N) to classical groups. We will talk about the precise formulations of the construction, as well as some examples and related problems. In particular, we will explain in examples that this construction is capable of recovering the local Vogan L-packet for classical groups.

#### **Qiutong Wang (Zhejiang University)**

**Title:** Counting irreducible representations of general linear groups and unitary groups

**Abstract:** In this talk, I will begin by briefly introducing the counting method developed by Dan Barbasch, Jia-Jun Ma, Binyong Sun, and Chen-Bo Zhu, which is based on the theory of coherent continuation representations. I will then explain how this method can be used to obtain results on the number of isomorphism classes of irreducible Casselman–Wallach representations of a real reductive group G with a given infinitesimal character and a given complex associated variety. These results are expressed in terms of certain combinatorial data known as painted Young diagrams and assigned Young diagrams.

#### **Zhicheng Wang (Jilin University)**

**Title:** Branching Problems over Finite Fields

**Abstract:** In representation theory, the branching problem examines how irreducible representations of Lie groups decompose when restricted to subgroups of smaller rank. Within this context, distinction problems and branching for generalized Gelfand-Graev representations (GGGRs) are two prominent cases of focus. In this presentation, I will discuss both cases over finite fields. First, I will present several results on distinction problems for symmetric spaces, including relevant aspects of Lusztig's multiplicity formula. Second, I will establish a general multiplicity formula for generalized Gelfand-Graev representations (GGGRs), which extends the multiplicity-one property of classical Gelfand-Graev representations. Furthermore, through explicit computations of these multiplicities for certain classical groups, I will demonstrate a duality phenomenon analogous to BZSV duality over finite fields.