微分算子的谱及其应用研讨会

——会议手册——

天元数学国际交流中心

2025 年 8 月 10 日-8 月 15 日

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天元数学国际交流中心

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组织委员会

刘爽（北京理工大学）

周茂林（南开大学）

楼元（上海交通大学）

会议安排

8 月 11 日		
时间	**内容**	**主持**
8:45-08:50	**开幕式**	楼元
8:50-09:40	报告人：**王凤雨** Title： Spectral Representations on Wasserstein Limits of Empirical Measures on Manifolds	
9:40-10:00	**茶歇**	
10:00-10:50	报告人：**梁兴** Title： Asymptotic behavior of spreading speeds of KPP equations in two-scale almost periodic media	楼元
10:50-11:40	报告人：**唐昊** Title：Some Results on Stochas/c Keller-Segel Systems	
11:40-14:00	**休息**	
14:00-14:50	报告人：**董昭** Title：ERGODIC AND MIXING PROPERTIEF THE 2D NAVIER-STOKES EQUATIONS WITH A DEGENERATE MULTIPLICATIVE GAUSSIAN NOISE	李芳
14:50-15:10	**茶歇**	
15:10-16:10	报告人：**周韬** Title： On the generalized principal eigenvalues and symmetry propagation of KPP equations	李芳
16:10-16:30	**茶歇**	
16:30-17:20	报告人：**亓维维** Title： Eigenvalue problems in noise driven multiscale dynamics	李芳

8 月 12 日		
时间	**内容**	**主持**
8:50-09:40	报告人：**麻希南** Title：非线性椭圆 k-Hessian 算子的几个研究	王治安
9:40-10:00	**茶歇**	
10:00-10:50	报告人：**王作勤** Title：On prescribing finite spectrum	王治安
10:50-11:40	报告人：**张海** Title ： In-gap interface modes in topological photonic/phononic structures	
11:40-14:00	**休息**	
14:00-14:50	报告人：**王治安** Title ： Global dynamics of a periodic diffusive consumer-resource model	王学锋
14:50-15:10	**茶歇**	
15:10-16:10	报告人：**王威** Title：Spiky Eigenfunctions and Wave Localization	王学锋
16:10-16:30	**茶歇**	
16:30-17:20	报告人：**白学利** Title ： NON-CONVERGENCE OF THE PRINCIPAL EIGENVALUE OF ELLIPTIC OPERATORS FOR LARGE ADVECTION	王学锋

8 月 13 日		
时间	**内容**	**主持**
8:50-09:40	报告人：**王学锋** Title ： Existence and nonexistence of stable patterns in semilinear nonlocal diffusion equations	李万同
9:40-10:00	**茶歇**	
10:00-10:50	报告人：**王智诚** Title：Optimization of the Principal Eigenvalue of a Cooperative Elliptic System with Drift	李万同
10:50-11:40	报告人：**姚若飞** Title： The location of the nodal line of the second Neumann eigenvalue in obtuse triangle	
11:40-14:00	**休息**	
14:00-14:50	**Free Discussion**	天元数学国际交流中心
14:50-15:10		
15:10-16:10		
16:10-16:30		
16:30-17:20		

8 月 14 日		
时间	**内容**	**主持**
8:50-09:40	报告人：**李万同** Title： Principal Eigenvalue of Nonlocal Operators and Applications	王智诚
9:40-10:00	**茶歇**	
10:00-10:50	报告人：**李芳** Title： Stability analysis of an SIS epidemic model in heterogeneous environment	王智诚
10:50-11:40	报告人：**陈洪葛** Title： Estimates of Dirichlet eigenvalues for homogeneous Hormander operators	
11:40-14:00	**休息**	
14:00-14:50	报告人：**储继峰** Title：Optimization problems for Sturm-Liouville problems	周茂林
14:50-15:10	**茶歇**	
15:10-16:10	报告人：**蒋凯** Title：丢番图误差的算法和理论	周茂林
16:10-16:30	**茶歇**	
16:30-17:20	报告人：**吴瑞军** Title：Continuity of weighted eigenpairs of Dirac operator	周茂林

摘要

王凤雨

（天津大学）

Spectral Representations on Wasserstein Limits of Empirical Measures on Manifolds

Abstract：Sharp convergence rates in Wasserstein distance are derived for empirical measures of diffusion processes on Riemannian manifolds, and the renormalization limits are explicitly formulated by using eigenvalues and eigenfunctions of the associated elliptic operator. For explosive diffusion processes, the convergence is described by conditional expectations.

梁兴

（中国科学技术大学）

Asymptotic behavior of spreading speeds of KPP equations in two-scale almost periodic media

Abstract：This work is concerned with the asymptotic behavior of the
spreading speeds of Fisher-KPP equations in rapidly and slowly varying
almost periodic settings respectively. Thanks to the variational formula for
the spreading speeds involving generalized principal eigenvalue, we are able
to concentrate on the homogenization problem of certain Hamilton-Jacobi
equations. We will present also the limits and the convergence rate of the
spreading speeds. At last, we will consider the effect on the spreading speeds
after adding a normal scale oscillation reaction term.

唐昊

（天津大学）

Some Results on Stochastic Keller-Segel Systems

Abstract: This talk explores recent advances in stochastic Keller-Segel systems driven by *pseudo-differential noise* (Stratonovich) and *multiplicative noise* (Itô). I will first introduce the local-in-time results, including existence, uniqueness, and blow-up criteria. The main focus will then shift to how these noise structures influence long-time behavior, particularly in different functional norms.

董昭

（中国科学院数学与系统科学研究院）

ERGODIC AND MIXING PROPERTIEF THE 2D NAVIER-STOKES EQUATIONS WITH A DEGENERATE MULTIPLICATIVE GAUSSIAN NOISE

Abstract： In this talk, we establish the ergodic and mixing properties of stochastic 2D Navier-Stokes equations driven by a highly degenerate multiplicative
 Gaussian noise. The noise can appear in as few as four directions, and its
 intensity depends on the solution. The case of additive Gaussian noise was
 previously treated by Hairer and Mattingly [Ann. of Math., 164(3):993 – 1032,
 2006]. To derive the ergodic and mixing properties in the present setting, we
 employ Malliavin calculus to establish the asymptotically strong Feller property. The primary challenge lies in proving the "invertibility" of the Malliavin matrix, which differs fundamentally from the additive case

周韬

（安徽大学）

On the generalized principal eigenvalues and symmetry propagation of KPP equations

Abstract： In this talk, I will briefly introduce the motivation for defining the generalized principal eigenvalue and its properties, then apply it to the study of the propagation symmetry of the KPP equation, including local and non-local equations. Additionally, some interesting differences and connections in the propagation of different equations will be presented.

亓维维

（中国科学院数学与系统科学研究院）

Eigenvalue problems in noise driven multiscale dynamics

Abstract：Real systems are often subject to noises due to internal uncertainties and complexity as well as external randomness. Even small noises can lead to intriguing multiscale dynamics including transient dynamics and long-term dynamics which can be captured by quasi-stationary distributions and stationary distributions, respectively. In the framework of randomly perturbed ODEs, (quasi-)stationary distributions turn out to be the principal eigenfunctions of the associated Fokker-Planck operator. The detailed asymptotic behaviors of (quasi-)stationary distributions in the zero noise limit would shed light on the dynamical mechanism of the noise driven multiscale dynamics. However, this analysis presents a singular limit, posing significant challenges. A powerful approach is to establish the large deviation principle (LDP) for (quasi-)stationary distributions. In this talk, I will present our recent results on the existence and regularity of the quasi-potential function in LDP, which have broad and impactful applications across various disciplines. Specifically, we will discuss its applications in non-equilibrium thermodynamics.

麻希南

（中国科学技术大学）

非线性椭圆 k-Hessian 算子的几个研究

摘要：首先回顾 k-Hessian 算子的基本性质，以及相关椭圆边值问题的可解性。k-hessian 算子的特征值问题从 80 年代由 Lions，汪徐家等开始研究，也会提及有关特征值等周估计的问题。最后陈述近来我们在 k-Hessian 算子 Green 函数的正则性工作。

王作勤

（中国科学技术大学）

On prescribing finite spectrum

Abstract：In 1980s Colin de Verdiere proved that on any closed manifold of dimension at least 3, one can construct a smooth metric with arbitrarily prescribed finite parts of eigenvalues. Later on Lohkamp showed that one can further prescribe the volume. In this talk, I will explain some backgrounds on this problem, and explain how to extend their results to Dirichlet eigenvalues and Robin eigenvalues on manifolds with boundary. This is based on joint works with He Xiang.

张海

（香港科技大学）

In-gap interface modes in topological photonic/phononic structures

Abstract：The development of topological insulators has provided a new avenue for creating interface modes (or edge modes) in photonic/phononic structures. Such created modes have a distinct property of being topologically protected and are stable to perturbations in certain classes. In this talk, we will report recent progress on the existence of in-gap interface modes in various topological photonic/phononic structures

王治安

（香港理工大学）

Global dynamics of a periodic diffusive consumer-resource model

Abstract： We consider a reaction-diffusion model describing the consumer-resource interactions, where the resource's input rate may be temporally periodic and spatially heterogeneous. By employing the parabolic comparison principle, method of super-lower solutions for the mixed-quasilinear monotone system, theory for asymptotically periodic systems, uniform persistence theory for infinite-dimensional dynamical systems, and principal eigenvalue theory, we classify the persistence and extinction dynamics of the consumer population in terms of dispersal rates and relaxation time classified by the mortality rate of the consumer. Furthermore, we derive the asymptotic profiles of positive periodic solutions as the resource's dispersal rate is sufficiently small or large. Our results elucidate how the consumer's mortality rate, the relaxation time, the spatiotemporal heterogeneity of the resource's input, and the dispersal rates affect the global dynamics of consumer and resource populations. In particular, our analytical results derive the following implications: (a) the resource's decay is the dominant factor that can prevent the resource abundance from blowing up; (b) the consumer’s mortality rate is a key factor determining the persistence and extinction for the consumer population; (c) the temporally homogeneous resource input may be more beneficial to the consumer's persistence than the temporally varying input when the consumer’s mortality is moderate.

王威

（中国科学院数学与系统科学研究院）

Spiky Eigenfunctions and Wave Localization

Abstract：Waves, ranging from electromagnetic and acoustic waves to mechanical vibrations, significantly shape the structure and dynamics of our world. In disordered systems, wave propagation is significantly hindered, impeding their mobility. Recent advancements in theoretical frameworks have introduced the concept of the 'localization landscape,' offering a promising approach to understanding wave localization in disordered media. This presentation will explore this innovative perspective, providing insights into localizations originating from disordered operators, and a new understanding of computing eigenpairs.

白学利

（西北工业大学）

NON-CONVERGENCE OF THE PRINCIPAL EIGENVALUE OF ELLIPTIC OPERATORS FOR LARGE ADVECTION

Abstract: In this talk, we mainly investigates the limit of the principal eigenvalue $\lambda(s)$ as the advection coefficient $s \to +\infty$ for an linear elliptic equation posed on a bounded domain under the homogeneous Neumann boundary
condition. In recent decades, numerous studies have focused on obtaining sharp
estimates for the limits of $\lambda(s)$ as $s \to \infty$. However, a more fundamental question remains open that whether the limit of $\lambda(s)$ always exists for an arbitrary vector field v as $s \to \infty$. In this paper, we present the first example of v for which $\lambda(s)$ does not converge as $s \to +\infty$. This is a joint work with Zhi-An Wang, Xin Xu, Kexin Zhang and Maolin Zhou.

王学锋

（香港中文大学（深圳））

Existence and nonexistence of stable patterns in semilinear nonlocal diffusion equations

Abstract: According to the classical results of Casten, Holland, and Matano regarding semilinear local diffusion equations on bounded domains with no-flux boundary condition, we know that stable patterns do not exist in convex domains, while they do emerge in dumbbell-shaped geometries, particularly when the kinetic term is bistable. Thus diffusion demolishes spikes, stipes and the likes. We ask what happens in the case of non-local diffusion.

 We recover the classical results for the nonlocal diffusion analogs, demonstrating the absence of stable smooth patterns in both one-dimensional intervals and multi-dimensional balls. In addition, we construct discontinuous, asymptotically stable patterns when the kinetic term is bistable. Our results reveal a significant principle: large nonlocal diffusion tends to destabilize patterns, whereas weak nonlocal diffusion stabilizes them, especially in cases with bistable kinetic terms. Importantly, the geometry of the domain appears to play a less critical role in this process of stabilization.

This is a joint work with Professors Fang Li and Xueli Bai.

王智诚

（兰州大学）

Optimization of the Principal Eigenvalue of a Cooperative Elliptic System with Drift

Abstract: In this talk, the optimization (minimization/maximization) of the principal eigenvalue of a cooperative elliptic system with drift is investigated, where the drift varies under an L^∞ constraint. We first show that the minimization and maximization of the principal eigenvalue are reachable and the associated drifts are unique, respectively,

then we obtain explicit expressions of minimal and maximal drifts for some special cases. In particular, the optimal principal eigenvalues are monotone in the amplitude of optimal drifts while the principal eigenvalue usually has no monotonicity result with respect to the amplitude of other drifts. These theoretical results have a wide range of applications in population dynamics, infectious disease prevention and control.

姚若飞

（华南理工大学）

The location of the nodal line of the second Neumann eigenvalue in obtuse triangle

Abstract: In this talk, we present a result concerning the location of the nodal line of the second eigenfunction for the Neumann problem in an obtuse triangle. This result resolves a conjecture proposed by Atar and Burdzy in Electronic Communications in Probability (2002).

李万同

（兰州大学）

Principal Eigenvalue of Nonlocal Operators and Applications

Abstract：In this talk, I will report some recent progress on principal eigenvalue of nonlocal operators and applications

李芳

（中山大学）

Stability analysis of an SIS epidemic model in heterogeneous environment

Abstract: In this talk, we study an SIS reaction-diffusion model in spatially heterogeneous environment proposed in L.J.S. Allen, B.M. Bolker, Y. Lou and A.L. Nevai, DCDS, 2008, where the existence and uniqueness of the endemic equilibrium are established and its stability is proposed as an open problem. However, till now, there is no progress in the stability analysis except for special cases with either equal diffusion coefficients or constant endemic equilibrium. In this talk, we demonstrate the first criterion in determining the stability of the non-constant endemic equilibrium with different diffusion coefficients. Thanks to this criterion, when one of the diffusion rates is small or large, the impact of spatial heterogeneity on the stability can be characterized based on the asymptotic behavior of the endemic equilibrium.

陈洪葛

（中国科学院精密测量科学与技术创新研究院）

Estimates of Dirichlet eigenvalues for homogeneous Hormander operators

Abstract: In this talk, we are concerned the Dirichlet eigenvalue problem for homogeneous H\"{o}rmander operators of the form $\triangle_{X}=\sum_{j=1}^{m}X_{j}^{2}$ on a bounded open domain in $\mathbb{R}^n$ containing the origin, where $X_1, X_2, \ldots, X_m$ are linearly independent smooth vector fields satisfying H\"{o}rmander's condition and a suitable homogeneity condition with respect to a family of non-isotropic dilations. Suppose that Ω is an open bounded domain in $\mathbb{R}^n$ containing the origin. We establish the explicit asymptotic behaviour $\lambda_k \approx k^{\frac{2}{Q_0}}(\ln k)^{-\frac{2d_0}{Q_0}}$ as $k \to +\infty$, where λ_k denotes the k-th Dirichlet eigenvalue of $\triangle_{X}$ on Ω, Q_0 is a positive rational number, and d_0 is a non-negative integer. Furthermore, we provide optimal bounds of index Q_0, which depend on the homogeneous dimension associated with the vector fields $X_1, X_2, \ldots, X_m$.

储继峰

（杭州师范大学）

Optimization problems for Sturm-Liouville problems

Abstract: We present some recent results on sharp bounds for eigenvalues and locations of nodes of the classical Sturm-Liouville problems.

蒋凯

（湘潭大学）

丢番图误差的算法和理论

摘要：数是数学研究的基石，亦是理解自然现象的基础。实数由零测度的有理数集和满测度的无理数集共同构成。由此推断，无理数起主导作用的体系理应更为普遍。然而，由于计算机难以存储和表示无理数，在对无理数起主导作用体系进行数值计算时会产生丢番图误差，即有理数逼近无理数之误差。这类误差会对计算结果的准确性可能起决定性影响。在本报告中，我们将分析丢番图误差机理及其对数值计算的影响，建立任意维丢番图频率准周期函数的逼近理论；进而提出能避免丢番图误差的新型算法--投影法和有限点恢复法，将高精度数值计算的范围从有理数域拓展到实数域，并利用无理数的遍历性和算术性质，建立算法的数学理论。最后，作为一个具体应用，我们将运用所发展的方法研究高维准周期薛定谔算子。

吴瑞军

（北京理工大学）

Continuity of weighted eigenpairs of Dirac operator

Abstract: Motivated by physical and geometric variational problems, we consider a weighted eigenvalue problem for Dirac operators on spin manifolds. Using a new minmax characterization of the weighted eigenvalues, we prove the complete continuity of eigenpairs, and also the continuity of the eigen-projectors. We will also discuss some applications.