Tianyuan Mathematics Research Centre (TYMC)

Seminar on Domain Theory and its Applications

2025.12.14 – 2025.12.20

We are hosting an academic seminar at the Tianyuan Mathematics Research Center (TYMC) from December 14 to 20, 2025. The event primarily gathers scholars worldwide whose research focuses on domain theory, non-Hausdorff topology, denotational semantics, topological algebra, logic, and related fields. The seminar will center on in-depth discussions of domain theory and its applications, while also encouraging researchers with diverse academic backgrounds to engage in further collaborative research. A total of 16 academic presentations are scheduled for the seminar, each lasting roughly one hour, with 50 minutes allocated to the formal presentation and 10 minutes reserved for questions and open discussion. In adherence to the core philosophy of the Center, which emphasizes extensive communication, free-ranging dialogue, and in-depth collaboration, the total duration of all presentations will be deliberately limited to no more than half of the overall event time.

1 Event Schedule:

Registration：2025.12.14 (Sunday)； Talks & Discussion：2025.12.15–19； Leave: 2025.12.20.
Venue：TYMC，Hua Luogeng Lecture Hall.

2 Important Notes:

1. This seminar is sponsored by the TYMC. Accommodation and meals for all participants will be covered by the Center, while other expenses shall be on the participants themselves.

2. The shuttle buses running between the TYMC and the airports/railway stations are available for free reservation. All participants are requested to register in advance.

3. The center is approximately a 40-minute drive from Gucheng Town Central Health Center, about a 1-hour drive from Yiliang County People's Hospital, and about a 1-hour drive from Shilin Yi Autonomous County People's Hospital. It is advisable to carry some essential medicines with you at all times. For medical emergencies, the center keeps a stock of com-

monly used medicines, including those for mosquito bites, 999 Ganmaoling Granules, Lianhua Qingwen Capsules, Yunnan Baiyao Spray, and bandages.

4. More information can be found at: http://tianyuan.amss.ac.cn/syyj/index.html

3 Organizers:

Seminar Applicant:

奚小勇/Xi Xiaoyong (盐城师范学院/Yancheng Teachers University)

Conveners:

寇辉/Kou Hui（四川大学/Sichuan University）

贾晓东/Jia Xiaodong (湖南大学/Hunan University)

Jean Goubault-Larrecq (University Paris-Saclay)

4 Invited speakers:

Matthew de Brecht （Kyoto University）

Jean Goubault-Larrecq （University Paris-Saclay）

Weng Kin Ho （Nanyang Technological University）

Tomáš Jakl （Czech Technical University in Prague）

Sebastian Melzer (New Mexico State University)

Hannes Thiel (Chalmers University of Technology and University of Gothenburg)

Dongsheng Zhao (Nanyang Technological University)

寇辉/Kou Hui（四川大学/Sichuan University）

李庆国/Li Qingguo (湖南大学/Hunan University)

吕振超/Lyu Zhenchao（四川大学/Sichuan University）

彭德奎/Peng Dekui（南京师范大学/Nanjing Normal University）

阮祥龙/Ruan Xianglong (陕西师范大学/Shannxi Normal University)

沈冲/Shen Chong (北京邮电大学/Beijing University of Posts and Telecommunications)

师维学/Shi Weixue (南京大学/Nanjing University)

徐晓泉/Xu Xiaoquan (闽南师范大学/Minnan Normal University)

张德学/Zhang Dexue (四川大学/Sichuan University)

天元数学国际交流中心 (TYMC)

2025 年 12 月 14 日

5 Program

12.15 (Monday)	
08:40–09:00	**Opening ceremony**
09:00–10:00	Li Qingguo **Some results in quasi-metric spaces**
10:00–10:30	**Tea break**
10:30–11:30	Weng Kin Ho **The Collatz Problem of Domain Theory: Is FS=RB?**
	Lunch
15:00–16:00	Jean Goubault-Larrecq **An introduction to semitopological barycentric algebras**
16:00–16:30	**Tea break**
16:30–17:30	Kou Hui **Compactness, Isbell topology and Function spaces**

12.16 (Tuesday)	
09:00–10:00	Xu Xiaoquan **A dichotomy theorem for Scott non-sober dcpos**
10:00–10:30	**Tea break**
10:30–11:30	Peng Dekui **Cardinal Spectra**
	Lunch
14:30–15:30	Ruan Xianglong **Representation and duality for CKL-algebras**
15:30–16:00	**Tea break**
16:00–17:00	Tomáš Jakl **Canonical and filter extensions of frames**
17:00–18:00	**Free discussion**

12.17 (Wednesday)	
09:00–10:00	Zhao Dongsheng **Poset models of topological spaces**
10:00–10:30	**Tea break**
10:30–11:30	Hannes Thiel **Domains arising in operator algebras**
	Lunch
14:30–18:00	**Free discussion**

12.18 (Thursday)	
09:00–10:00	Zhang Dexue **d-Lattices and d-Spectral Spaces**
10:00–10:30	**Tea break**
10:30–11:30	Shi Weixue **An introduction to generalized ordered topological spaces**
	Lunch
14:30–15:30	Lyu Zhenchao **The Scott space of lattice of closed subsets**
15:30–16:00	**Tea break**
16:00–17:00	Shen Chong **Some Results on Poset Models and (Co)Reflectivity of T_0 Spaces**
17:00–18:00	**Free discussion**

12.19 (Friday)	
09:00–10:00	Matthew de Brecht **Effective subcategories of quasi-Polish spaces**
10:00–10:30	**Tea break**
10:30–11:30	Sebastian Melzer **Locally compact and sober, but not quite enough points**
	Lunch
14:30–18:00	**Free discussion**

Seminar on Domain Theory and its Applications

2025.12.14–12.20, Tianyuan Mathematics Research Centre

Li Qingguo: Some results in quasi-metric spaces.

In this report, we will discuss the characterization of three important separation properties in non-Hausdorff topology: soberness, well-filteredness, and being d-spaces in quasimetric spaces; the characterization of various forms of completeness in quasimetric spaces; sufficient conditions for the equality of the open ball topology and the Scott topology on formal ball spaces; and finally, we will discuss properties of the function spaces consisting of all α-Lipschitz functions and all α-Lipschitz continuous functions on quasimetric spaces, respectively.

Ho Weng Kin: The Collatz Problem of Domain Theory: Is FS=RB?

In domain theory, finitely separated (FS) domains occupy a central place as they form one of the maximal Cartesian Closed subcategories of pointed domains. It is well-known that algebraic FS are exactly the bifinite domains, and this immediately raises the question: Are FS-domains exactly the same thing as retracts of bifinite domains? Domains of this latter form are referred to as RB-domains. The guiding analogy comes from the classical fact that every continuous domain can be realized as a retract of an algebraic domain, suggesting that a similar correspondence might hold in the context of FS-domains. This talk provides an accessible introduction to the problem of whether FS = RB. I will review the definitions of FS-domains, bifinite domains, and RB-domains, illustrating each with examples. We will then trace the motivations behind the conjecture, survey the latest known results, and discuss why the problem has remained resistant to solution. While this problem may seem to be a boondoggle as the title suggests, I hope to articulate the technical challenges explicitly and encourage the building of new tools to bring it down.

Jean Goubault-Larrecq: An introduction to semitopological barycentric algebras.

I will give an introduction to barycentric algebras, the plain, the preordered and the (semi)topological. Those include all convex subsets of (plain, preordered, semitopological) cones, but there are a few more. Then I will turn to pointed barycentric algebras, the preordered and the (semi)topological. The leading examples are spaces of probability, and subprobability valuations (or measures).

Kou Hui: Compactness, Isbell topology and Function spaces.

The main goal of this talk, is to commemorate Liu Yingming and Liang Jihua, who led the Chinese scholars to start the study of domain theory in 1990's. We will introduce the early remarkable results of function spaces obtained by Liu Yingming and Liang Jihua, and several follow-up results closely related compactness and Isbell topology.

Xu Xiaoquan: A dichotomy theorem for Scott non-sober dcpos.

In this talk, we investigate conditions under which the Scott topology of a dcpo is non-sober. We prove that for a T_0-space X, if the lattice of all closed subsets of X is a Heyting algebra and X with the specialization order is dual Noetherian, then X is a B-space, that is, the lattice of all open subsets is a strong algebraic lattice. As two corollaries, we get that for a d-space X, if the lattice of all closed subsets of X is a Heyting algebra and X with the specialization order is dual Noetherian, then X with the specialization order is an algebraic domain and the topology on X is the Scott topology, and if the Scott space ΣP of a dcpo P is non-sober, then either P is not meet-continuous or P is not dual Noetherian. The complete Heyting algebra with non-sober Scott topology, given by Xu, Xi and Zhao, is not dual Noetherian, and five other famous dcpos with non-sober Scott topologies are not meet-continuous. Finally, we point out that the following two questions may be quite difficult to be solved: Is there a countable frame whose Scott space is non-sober? Is there a countable spatial frame whose Scott space is non-sober?

Peng Dekui: Cardinal Spectra.

We introduce the notion of the density spectrum of a dense subgroup of a topological group G, denoted by $dd^*(G)$, defined as the set of all cardinals that can occur as the density of a dense subgroup of G. We show that $dd^*(G)$ forms precisely the interval between the density of G and the weight of G. We also introduce another type of cardinal spectrum, namely the weight spectrum of quotient groups of a topological group, and we prove that this set can be highly arbitrary when G is precompact.

Ruan Xianglong: Representation and duality for CKL-algebras.

The main purpose of this talk is to investigate the topological representation of CKL-algebras. We prove that every CKL-algebra A can embed into the L-algebra of up-sets of the poset of its prime ideals if and only if A is self-distributive. Based on this, we construct a duality between the category of self-distributive L-algebras and the category of H-spaces, and restrict this duality to obtain the Stone duality for Boolean algebras. Further, we prove that the category of prime KL-algebras is isomorphic to the category of posets with greatest elements, and apply this result to establish the topological duality for posets.

Tomáš Jakl: Canonical and filter extensions of frames.

In this talk I will review the theory of canonical extensions of lattices and explain how it relates to Priestley duality and my earlier work on canonical extensions of frames. Following this I will present a joint work with Anna Laura Suarez which generalises this and explains how many recent frame-theoretic constructions are naturally understood as versions of canonical extensions of frames (which we call filter extensions).

Zhao Dongsheng: Poset models of topological spaces.

A poset model of a topological space X is a poset P such that the set $\max(P)$ of all maximal points of P with the relative Scott topology on P (called the maximal point space of P) is homeomorphic to X. Although the Scott spaces of posets are just T_0 in general, the maximal point spaces of posets are quite general: they are exactly the T_1 spaces. In this talk, we will present a survey on the maximal point spaces of posets. The talk will include the following parts:

(i) the poset of all closed intervals of real numbers and some special features;

(ii) models of metric spaces;

(iii) Xi-Zhao model and main properties;

(iv) spaces that have a bounded complete dcpo model;

(v) models via other topologies;

(vi) some problems.

Hannes Thiel: Domains arising in operator algebras.

Continuous dcpos arise naturally in the study of operator algebras through the comparison theory of positive elements, extending the classical Murray-von Neumann comparison for projections. These domains come equipped with a natural semigroup structure induced by orthogonal addition, and the resulting domain semigroups play a central role in the structural and classification theory of C*-algebras.

I will outline the construction of these domains, highlight some key examples, and present a recent structure theorem showing that, under natural hypotheses, such domain semigroups are inf-semilattices.

Zhang Dexue: d-Lattices and d-Spectral Spaces.

Klinke, Jung and Moshier proposed a notion of d-lattices in 2011, which is an extension of that of d-frames introduced by Jung and Moshier in 2006 as an algebraic dual of bitopological spaces. Roughly speaking, a d-lattice is a structured distributive lattice, the structure consists of a complementary pair of elements and two subsets of the lattice, called the consistency predicate and the totality predicate. We talk about the relationship among distributive lattices, d-lattices and bitopological spaces. In particular, we show that spectra of d-lattices are precisely d-spectral bitopological spaces and that the category of d-lattices contains the category of distributive lattices as a simultaneously reflective and coreflective full subcategory.

Shi Weixue: An introduction to generalized ordered topological spaces.

Recent twenty years some articles were published in whose articles authors investigate problems in generalized ordered spaces by using Domain theory. The scholars who are interested in reading those articles maybe need to know something about generalized ordered spaces. In this talk, I will introduce the definitions, basic properties and some examples on generalized ordered topological spaces.

Lyu Zhenchao: The Scott space of lattice of closed subsets.

We present several equivalent conditions of the continuity of the supremum function $\Sigma C(X) \times \Sigma C(X) \to \Sigma C(X)$ under mild assumptions, where $C(X)$ denotes the lattice of closed subsets of a T_0 topological space. We also provide an example of a non-monotone determined space X such that $\eta = \lambda x.{\downarrow}x : X \to \Sigma C(X)$ is continuous. Additionally, we show that a T_0 space is quasicontinuous (quasialgebraic) iff the lattice of its closed subsets is a quasicontinuous (quasialgebraic) domain by using n-approximation. Furthermore, we provide a necessary condition for when a topological space possesses a Scott completion. This allows us to give more examples which do not have Scott completions.

Shen Chong: Some Results on Poset Models and (Co)Reflectivity of T_0 Spaces.

Poset models establish a fundamental link between partially ordered sets and Hausdorff spaces. Specifically, a poset model of a topological space X is a poset P such that X is homeomorphic to the set $\mathbf{Max}(P)$ of maximal elements of P, equipped with the subspace topology induced by the Scott topology on P. It is known that every T_1 space admits a dcpo model, commonly referred to as the Xi–Zhao dcpo model. In this report, we first introduce a novel dcpo model for first-countable T_1 spaces, which features a simplified order structure compared to the original Xi–Zhao construction. We also review the T_0 variant of the Xi–Zhao model, which shows that every d-space can be embedded as a saturated subspace of a dcpo. Consequently, the reflective hull of the category of dcpos endowed

with the Scott topology in $\mathbf{TOP_0}$ coincides with the category of d-spaces. This result suggests some potential connections between reflective hulls and poset models in the T_0 setting. Finally, we present several new results on the coreflectivity of topological properties in the category of T_0 spaces.

Matthew de Brecht: Effective subcategories of quasi-Polish spaces.

In previous work, we constructed the category of quasi-Polish spaces as a represented space, and showed that limits, coproducts, and standard powerspace monads are computable in the sense of Type Two Theory of Effectivity (TTE).

In this talk, we show that some important sub-categories of quasi-Polish spaces (in particular the subcategories of overt discrete quasi-Polish spaces and compact Hausdorff quasi-Polish spaces) can be constructed as effective quasi-Polish spaces (i.e., effective internal categories of the category of quasi-Polish spaces).

To show that the constructions are natural, we show that Stone duality (for both objects and morphisms) is computable, in the sense that the dual contravariant functors and the natural transformations demonstrating their adjointness are computable.

(This talk was first given at ALC 2025)

Sebastian Melzer: Locally compact and sober, but not quite enough points.

Frames abstract lattices of open sets of topological spaces. Distributive continuous lattices are frames, and they are spatial by Hofmann-Lawson duality: they arise as the opens of locally compact sober spaces. MT-algebras instead generalize powerset interior algebras, capturing both frames and all spaces in a single algebraic setting. Motivated by the situation for frames, it was conjectured that locally compact sober MT-algebras are spatial. We show that this is not the case.

This is joint work with Guram Bezhanishvili, Ranjitha Raviprakash, and Anna Laura Suarez.