Analysis and Numerics in Grating Diffraction Problems

Dec 28, 2025 - Jan 2, 2026

Tianyuan Mathematical Research Center
Yunnan, China

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2025. 12. 28 星期日 (Sunday）

18:00-20:00 自助餐（buffet）

会议日程（Conference Schedule）

2025.12.29 星期一 (Monday）			
09:10-09:30	**欢迎致辞 (Welcome Speech)**		
时间 (time)	报告人(Speaker)	题目 (Title of talk)	主持人 (Chair)
09:30-10:10	Lifeng Li	Problem of grating diffraction involving edge hypersingularity	Gang Bao
10:10-10:30	休息 (Break)		
10:30-11:10	Jiguang Sun	Finite element methods for nonlinear eigenvalue problems of periodic structures	Guanghui Hu
11:10-14:30	午餐 (Lunch)		
14:30-15:10	Buyang Li	Computing rough solutions of nonlinear wave equations	Haijun Wu
15:10-15:50	Ruming Zhang	The limiting absorption principle for Helmhotz equations in n-dimensional periodic structures	Haiwen Zhang
15:50-16:10	休息 (Break)		
16:10-16:50	Ping Liu	A mathematical theory of computational resolution limit and super-resolution	Hai Zhang
16:50-17:30	自由讨论 (Free discussion)		

2025.12.30 星期二 (Tuesday)			
时间 (time)	报告人(Speaker)	题目 (Title of talk)	主持人 (Chair)
09:30-10:10	Ya Yan Lu	Asymptotically circularly polarized bound states in the continuum	Lifeng Li
10:10-10:30	休息 (Break)		
10:30-11:10	Luiz Faria	TBD	Wangtao Lu
11:10-14:30	午餐 (Lunch)		
14:30-15:10	Yuliang Wang	Inverse scattering and super-resolution imaging of diffraction gratings via transformed field expansion	Junliang Lv
15:10-15:50	Tao Yin	Uniformly-highly accurate PML-BIE method for scattering by periodic arrays of obstacles	Huangxin Chen
15:50-16:10	休息 (Break)		
16:10-16:50	Fuhao Liu	An efficient numerical method for simulating two-dimensional non-periodic metasurfaces	Ping Liu
16:50-17:30	自由讨论 (free discussion)		

2025.12.31 星期三 (Wednesday）			
时间 (time)	报告人(Speaker)	题目 (Title of talk)	主持人 (Chair)
09:30-10:10	Haijun Wu	TBD	Buyang Li
10:10-10:30	休息 (Break)		
10:30-11:10	Junliang Lv	Uniqueness and numerical method for phaseless inverse diffraction grating problem	Lei Zhang
11:10-12:30	午餐 (Lunch)		

2026.1.1 星期四（Thursday）			
时间 (time)	报告人(Speaker)	题目（Title of talk)	主持人 (Chair)
09:30-10:10	Jun Lai	A singularity guided Nystrom algorithm for elasticity in cornered domains	Jiguang Sun
10:10-10:30	休息（Break)		
10:30-11:10	Hai Zhang	Anomalous scattering of light by subwavelength structures in metallic slabs	Jun Lai
11:10-15:00	午餐（Lunch)		
15:00-15:40	Haiwen Zhang	Asymptotics of the solution to the rough surface scattering problem and application to the inverse problem	Tao Yin
15:40-16:00	休息（Break)		
16:00-16:40	Jiaxin Zhou	Local robustness of bound states in the continuum revealed by the scattering matrix eigenvector evolution	Xiaokai Yuan
16:40-17:30	自由讨论（Free discussion)		

2026.1.2 星期五(Friday）			
时间 (time)	报告人(Speaker)	题目（Title of talk)	主持人 (Chair)
09:30-10:10	Long Meng	Quasicrystalline systems from a semiclassical analysis perspective	Xue Jiang
10:10-10:30	休息（Break)		
10:30-11:10	Xiaoxu Xu	Uniqueness in inverse diffraction grating problems with infinitely many plane waves at a fixed frequency	Xiaoli Liu
11:10-14:30	午餐（Lunch)		

附件 1 (Attachment 1)　报告信息（按姓氏拼音首字母排序）
Report information (Sort by phonetic initials of last names)

A singularity guided Nystrom algorithm for elasticity in cornered domains

Jun Lai, Zhejiang University

Abstract: In this talk, we give a comprehensive analytical and numerical framework for boundary integral equations (BIEs) of the 2D Lame system on cornered domains. Using local Mellin analysis on a wedge, we obtain a factorizable characteristic equation for the singular exponents of the boundary densities. The Fredholm well-posedness of the BIEs on cornered domains is proved in weighted Sobolev spaces. We further construct an explicit density-to-Taylor mapping for the BIE. Based on these results, we propose a singularity guided Nystrom (SGN) scheme to numerically solve BIEs on cornered domains. Numerical experiments across various cornered geometries demonstrate that SGN obtains higher order accuracy than uniform Nystrom method and reveal a crowding-limited regime for domains with re-entrant angles.

Computing rough solutions of nonlinear wave equations

Buyang Li, The Hong Kong Polytechnic University

Abstract: We present a high-frequency recovered low-regularity integratorfor computing rough and possibly discontinuous solutions of the nonlinear wave equation. The proposed method, with high-frequency recovery techniques, can significantly improve the accurayc of the numerical solutions. Rigorous analysis is presented for the convergence rates of the proposed method in approximating solutions such that (u,∂_tu) \in C([0,T]; H^γ × H^{γ-1}) for $\gamma\in(0,1]$. In particular, the proposed method is proved to have almost first-order convergence for computing discontinuous solutions of bounded variation. Numerical examples are presented in both one and two dimensions to illustrate the advantages of the proposed method in improving the accuracy in approximating rough and discontinuous solutions of the semilinear wave equation.

Problem of grating diffraction involving edge hypersingularity

Lifeng Li, Tsinghua University

Abstract: Consider the following simple grating problem.

Problem. A grating consists of an array of infinitely long square rods suspended in air (refractive index = 1), and periodically spaced in a horizontal direction. One side of the square rod is parallel to the horizontal plane. A TM polarized (magnetic field vector parallel to the rods) plane wave of wavelength λ is vertically incident on the grating. The period of the grating is $3\lambda/2$. The permeability and permittivity of the rod are $\mu = 1$ and $\varepsilon = -2$, respectively. Find the diffraction efficiencies of all propagating orders of this grating. Within the framework of electrodynamics of continuous media, the above problem is physically well-defined. Is it also mathematically well-defined? If not, why not? If yes, what are the diffraction efficiencies? Any rigorous numerical method can be used. I will discuss some physical, mathematical, and numerical issues related to this problem.

An efficient numerical method for simulating two-dimensional non-periodic metasurfaces

Fuhao Liu, City University of Hong Kong

Abstract: We present a Neumann-to-Dirichlet (NtD) operator-based method for efficiently simulating wave scattering by large non-periodic two-dimensional metasurfaces. This approach is particularly designed for structures comprising a huge number of unit cells, yet only a small number of distinct unit cell types. For each type of unit cells, a compact NtD operator, which maps the normal derivative of the wave field to the field itself on the boundary, is precomputed using the finite element method. By enforcing field continuity across all unit cell interfaces, a reduced global linear system is constructed for the interfacial normal derivatives. Solving this system allows for the subsequent reconstruction of the wave field within each subdomain, avoiding solving the entire scattering problem directly. This approach drastically reduces the total number of unknowns. Numerical examples demonstrate that the method maintains high accuracy while offering significant advantages in both computational time and memory usage compared to the classical full-domain finite element method, making it particularly suited for the analysis of large metasurfaces.

A mathematical theory of computational resolution limit and super-resolution

Ping Liu, Zhejiang University

Abstract: Due to the physical nature of wave propagation and diffraction, there is a fundamental diffraction barrier in optical imaging systems which is called the diffraction limit or resolution

limit. Rayleigh investigated this problem and formulated the well-known Rayleigh limit. However, the Rayleigh limit is empirical and only considers the resolving ability of the human visual system. On the other hand, resolving sources separated below the Rayleigh limit to achieve so-called "super-resolution" has been demonstrated in many numerical experiments.

In this talk, we will propose a new concept "computational resolution limit" which reveals the fundamental limits in super-resolving the number and locations of point sources from a data-processing point of view. We will quantitatively characterize the computational resolution limits by the signal-to-noise ratio, the sparsity of sources, and the cutoff frequency of the imaging system. As a direct consequence, it is demonstrated that l_0 optimization achieves the optimal order resolution in solving super-resolution problems. For the case of resolving two point sources, the resolution estimate is improved to an exact formula. We will also propose an optimal algorithm to distinguish images generated by single or two point sources. Generalization of our results to the imaging of positive sources, imaging of multi-cluster sources, and imaging in multi-dimensional spaces will be briefly discussed as well.

Uniqueness and numerical method for phaseless inverse diffraction grating problem

Junliang Lv, Jilin University

Abstract: In this report, I will introduce some results on uniqueness and numerical methods of identifying a smooth grating profile with a mixed or transmission boundary condition from phaseless data. The existing uniqueness result requires the measured data to be in a bounded domain. To break this restriction, we design an incident system consisting of the superposition of point sources to reduce the measurement data from a bounded domain to a line above the grating profile. We derive reciprocity relations for point sources, diffracted fields, and total fields, respectively. Based on Rayleigh's expansion and reciprocity relation of the total field, a grating profile with a mixed or transmission boundary condition can be uniquely determined from the phaseless total field data. An iterative algorithm is proposed to recover the Fourier modes of grating profiles at a fixed wavenumber. Some numerical examples are presented to verify the correctness of theoretical results and to show the effectiveness of our numerical algorithm.

Quasicrystalline systems from a semiclassical analysis perspective

Long Meng, Zhejiang University

Abstract: The computation of electronic structures in crystalline materials is one of the most challenging problems in condensed matter physics. Through the Bloch transform, calculations for periodic crystals can be transformed into computations for Bloch quasi-electrons. This theory explains conductors, semiconductors, and insulators. By breaking the periodicity of crystals, quasicrystalline systems are obtained. Quasicrystalline systems exhibit extraordinary physical phenomena such as superconductivity, Anderson localization, and flat bands, which have attracted considerable attention in modern physics. We find that the behavior of Bloch quasi-electrons in quasicrystals can be characterized by pseudo-differential operators. Consequently, we introduce a set of semiclassical analysis theories based on the Bloch transform, providing new theoretical support for many of the remarkable phenomena in quasicrystalline systems.

Finite element methods for nonlinear eigenvalue problems of periodic structures

Jiguang Sun, Michigan Technological University

Abstract: We review the abstract approximation theory for eigenvalue problems of holomorphic Fredholm operator-valued functions. The associated error estimates rely on the notion of regular convergence for the discrete approximation operators. We apply this framework to two nonlinear eigenvalue problems: the computation of band-structure diagrams for dispersive photonic crystals and the determination of scattering resonances in metallic grating structures with subwavelength apertures. For both problems, we establish finite element error estimates and present numerical examples that demonstrate the effectiveness of the framework.

Inverse scattering and super-resolution imaging of diffraction gratings via transformed field expansion

Yuliang Wang, Wenzhou-Kean University

Abstract: This presentation reviews a unified computational framework for the inverse scattering of diffraction gratings based on the Transformed Field Expansion (TFE) method. We begin by establishing the mathematical foundation using the Helmholtz equation for 1D surfaces, demonstrating how coordinate transformations yield efficient power series solutions. The methodology is subsequently extended to the complex regime of elastic waves (Navier equation) for both 1D and biperiodic structures, backed by rigorous convergence analysis. Crucially, by utilizing near-field data, this framework naturally overcomes the diffraction limit through the recovery of evanescent wave components. We further explore advanced imaging configurations—specifically high refractive index slabs and negative index "superlenses"—designed to enhance this super-resolution capability. These setups amplify evanescent waves, allowing for the stable reconstruction of subwavelength details even in challenging regimes. Numerical results will be presented to validate the method's stability and accuracy.

Uniqueness in inverse diffraction grating problems with infinitely many plane waves at a fixed frequency

Xiaoxu Xu, Xi'an Jiaotong University

Abstract: This talk is concerned with uniqueness of solution of the inverse diffraction by problems by a periodic curve with Dirichlet boundary condition in two dimensions. It is proved that the periodic curve can be uniquely determined by the near-field measurement data corresponding to infinitely many incident plane waves with distinct directions at a fixed frequency. Our proof is based on Schiffer's idea which consists of two ingredients: (i) the total fields for incident plane waves with distinct directions are linearly independent, and (ii) for a fixed wave number there exist only finitely many linearly independent Dirichlet eigenfunctions in a bounded domain or in a closed waveguide under additional assumptions on the waveguide boundary. Based on the Rayleigh expansion, we prove that the phased near-field data can be uniquely determined by the phaseless near-field data in a bounded domain, with the exception of a finite set of incident angles. Such a phase retrieval result leads to a new uniqueness result for the inverse grating diffraction problem with phaseless near-field data at a fixed frequency. Since the incident direction determines the quasi-periodicity of the boundary value problem, our inverse issues are different from the existing results of [F. Hettlich and A. Kirsch, Inverse Problems, 13 (1997), pp. 351–361], where fixed-direction plane waves at multiple frequencies were considered. This talk is based on a joint work with Guanghui Hu, Haiwen Zhang, and Bo Zhang.

Uniformly-highly accurate PML-BIE method for scattering by periodic arrays of obstacles

TaoYin, Chinese Academy of Sciences

Abstract: This talk will present a novel frequency-robust perfectly matched layer (PML)

boundary integral equation (BIE) method for solving the acoustic scattering problems involving periodic arrays of obstacles. A direct application of PML-BIE techniques to periodic structures experiences difficulties near RW anomalies. We propose a modified PML-BIE method that combines the PML technique with a finite-mode correction, ensuring both high accuracy and robustness at and around RW anomalies. The exponential convergence of the PML-truncated boundary integral operators and several numerical examples validating the efficiency and performance of the proposed method will be given.

Anomalous scattering of light by subwavelength structures in metallic slabs

Hai Zhang, The Hong Kong University of Science and Technology

Abstract: Since the discovery of the extraordinary optical transmission through nanohole arrays in metallic films by Ebbesen, a wealth of research has been sparked in the experimental and theoretical investigation of the transmission enhancement in subwavelength nanostructures. In this talk, using two-dimensional periodic slits as a prototype, I will present mathematical studies of the transmission enhancement in the subwavelength structures. Based upon the layer potential technique, asymptotic analysis, different types of enhancement mechanism are unveiled, which includes Fabry-Perot resonance, surface modes, and Fano resonances.

Asymptotics of the solution to the rough surface scattering problem and application to the inverse problem

Haiwen Zhang, Chinese Academy of Sciences

Abstract: This talk is concerned with direct and inverse scattering by an unbounded rough surface with the Dirichlet boundary condition in two dimensions. We study the asymptotics of the solution to the considered scattering problem with the boundary data in a weighted space of continuous functions. Based on this, we develop a direct imaging method for recovering the unbounded rough surface from the scattered-field data generated by incident point sources. Numerical experiments are carried out to demonstrate the feasibility and robustness of our algorithms.

Local robustness of bound states in the continuum revealed by the scattering matrix eigenvector evolution

Jiaxin Zhou, City University of Hong Kong

Abstract: Bound states in the continuum (BICs) are localized states that coexist with a continuum of propagating fields. Perturbations that destroy a BIC typically generate ultra-strong resonances, enabling diverse applications in photonics. We investigates the local robustness of BICs in a two-dimensional periodic structure for E-polarized, quasi-periodic fields. Using the implicit function theorem, we demonstrate that a simple BIC transforms continuously into a propagating field as the structural parameters vary in a neighborhood, with its corresponding frequency adjusting accordingly. Furthermore, the incident coefficients of the field persist as an eigenvector of the scattering matrix. By defining a map $\mathcal{P}$ from the structural parameters to these incident coefficients, BICs correspond precisely to zeros of $\mathcal{P}$. If such a zero is isolated and the domain and range dimensions of $\mathcal{P}$ match, the BIC can be related to the winding number (or mapping degree) of $\mathcal{P}$ in its neighborhood. We therefore define a BIC as locally robust with respect to the given parameters if it is associated with a nonzero winding number. This provides apractical numerical criterion for detecting and confirming locally robust BICs by computing the winding number of $\mathcal{P}$.

The limiting absorption principle for Helmhotz equations in n-dimensional periodic structures

Ruming Zhang, Technische Universität Berlin

Abstract: The limiting absorption principle (LAP) is a widely used tool to study elliptic equations. However, when the domain is unbounded and periodic, it becomes particularly challenging in particular when the structure is periodic in more than one dimensions. Therefore, fundamental improvement is necessary. In this talk, I will introduce a complete new approach, which is called a directional spectral analysis (DSA), to formulate the solution of the Helmholtz equations with higher dimensional periodic coefficients. This new approach efficiently formulate the LAP solution with respect to directions.

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