天元數學國際交流中心
Tianyuan Mathematics Research Center

Workshop on Combinatorics, Algebra and Geometry

10 May 2026 — 15 May 2026

Conference Handbook

Organizers

Beifang Chen（The Hong Kong University of Science and Technology）

Peter L. Guo (Nankai University)

Arthur L.B. Yang（Nankai Universit）

SCHEDULE OF WORKSHOP ON COMBINATORICS, ALGEBRA AND GEOMETRY

May 10			
All Day	Registration		
May 11			
8:20-8:30	Open Ceremony		
	TITLE	SPEAKER	HOST
8:30-9:10	Hyperplane arrangements induced from polytopes	Beifang Chen	Arhtur L.B. Yang
9:10-9:50	Convolution-type identity of geometric semilattice	Suijie Wang	
9:50-10:20	Break / Discussion		
10:20-11:00	Chern classes of open projected Richardson varieties and of affine Schubert cells	Changlong Zhong	Guoce Xin
11:00-11:40	Maximal bumpless pipedreams of double Grothendieck polynomials	Sophie C.C. Sun	
11:40-	Lunch		
14:30-15:10	有理凸多胞形格点计数的组合方法及其进展	Guoce Xin	Beifang Chen
15:10-15:50	Combinatorial algebraic geometry: polytopes, fans, and toric varieties	Tao Gui	
15:50-16:20	Break / Discussion		
16:20-17:00	Graph positivities	David G.L. Wang	Zhicong Lin
17:00-17:40	Hyperplane arrangement, tope graph and magnitude	Ye Liu	
17:40-	Dinner		

May 12			
	TITLE	SPEAKER	HOST
9:00-9:40	The intersection of enumerative combinatorics with coding theory, mathematical biology, and number theory	Zhicong Lin	Biao Zhang
9:40-10:20	Monomial expansion of modified Macdonald polynomials	Emma Yu Jin	
10:20-10:50	Break / Discussion		
10:50-11:30	On factorization of matrix of Kazhdan-Lusztig polynomials	Aritra Bhattacharya	Zhuowei Lin
11:30-	Lunch		
15:00-15:40	The peak algebra in noncommuting variables	Shu Xiao LI	Alice L.L. Gao
15:40-16:20	How to use deep learning to connect continuous graph invariants and Stanley's e-positivity of graphs	Farid Aliniaeifard	Alice L.L. Gao
16:20-17:30	Break / Discussion		
17:30-	Dinner		

May 13			
	TITLE	SPEAKER	HOST
8:30-9:10	Bivariate Chow polynomials for ordered matroids	Alice L.L. Gao	Emma Yu Jin
9:10-9:50	Recursive formulas for the polymatroid Tutte polynomial	Xiaxia Guan	
9:50-10:20	Break / Discussion		
10:20-11:00	Characteristic quasi-polynomials of integral arrangements	Houshan Fu	Suijie Wang
11:00-11:40	Ehrhart quasi-polynomials of rational polytopes by real dilations	Ying Cao	
11:40-	Lunch		
15:00-17:30	Break / Discussion		
17:30	Dinner		

May 14			
	TITLE	SPEAKER	HOST
8:30-9:10	A proof of the multi-component q-Baker--Forrester conjecture	Yue Zhou	Matthew H.Y.
9:10-9:50	Renormalization of quasisymmetric functions and its applications to multiple zeta values	Houyi Yu	
9:50-10:20	Break / Discussion		
10:20-11:00	Equivariant geometry of clans	Neil J.Y. Fan	Peter L. Guo
11:00-11:40	Positivity of double Schubert polynomials	Rui Xiong	
11:40-	Lunch		
15:00-17:30	Break / Discussion		
17:30	Dinner		

May 15	
All Day	Departure

Abstract

How to use deep learning to connect continuous graph invariants and Stanley's e-positivity of graphs

Farid Aliniaeifard Shandong University

We first show the capabilities of AI in mathematics by considering different philosophies of mathematics and their relationship with AI.

Then we use attribution techniques and exploratory data analysis to give sufficient conditions for a given statement and make conjectures. As a demonstration, we apply this process to Stanley' s problem of e-positivity of graphs. Guided by AI, we rediscover that one sufficient condition for a graph to be e-positive is that it is co-triangle-free, and that the number of claws is the most important factor for e-positivity. Based on the most important factors identified in the saliency map analysis of neural networks, we suggest that the classification of e-positive graphs is more likely related to continuous graph invariants rather than discrete ones, and we make three conjectures connecting continuous graph invariants with e-positivity.

On factorization of matrix of Kazhdan-Lusztig polynomials

Aritra Bhattacharya Peking University

The famous Kazhdan-Lusztig positivity states that the transition matrix between the canonical basis and the standard basis of the Hecke algebra (of a Weyl group) is a matrix with polynomial entries with non-negative integer coefficients. In this talk we will describe a factorization of this positivity via certain intermediate bases called the hybrid bases. We will also present a reformulation of the statements in terms of a natural restriction map on Hecke algebras. This framework also subsumes the positivity of parabolic Kazhdan-Lusztig polynomials corresponding to the sign representations. This is joint work with Ashish Mishra and Shraddha Srivastava.

Ehrhart quasi-polynomials of rational polytopes by real dilations

Ying Cao The Hong Kong University of Science and Technology

This talk is to study the Ehrhart function $L(P,t)$ of a rational n-polytope P, defined as the number of lattice points of dilated polytopes tP with real numbers $t \geq 0$. It turns out that $L(P,t)$ is a quasi-polynomial of real variable t in the sense that

$$L(P,t) = \sum_{k=0}^{n} c_k(P,t)t^k, \quad t \geq 0,$$

where $c_k(P,t)$ are periodic piecewise polynomials of degree $n-k$ if P contains the origin, and are periodic functions vanishing almost everywhere otherwise. When P is a rational simplex σ, the coefficient functions $c_k(\sigma,t)$ are given explicitly in terms of vertex information of the simplex σ. Moreover, the reciprocity law still holds.

Hyperplane arrangements induced from polytopes

Beifang Chen The Hong Kong University of Science and Technology

Equivariant geometry of clans

Neil J.Y. Fan Sichuan University

We relate bumpless pipe dreams on partitions to the equivariant geometry of $(GL_p \times GL_q)$-orbits over the flag variety Fl_{p+q} parametrized by (p,q)-clans. In particular, this gives a combinatorial formula for the equivariant Schubert expansion of the $(GL_p \times GL_q)$-orbit closure, which resolves an open problem of Wyser and Yong.

Characteristic Quasi-polynomials of Integral Arrangements

Houshan Fu Guangzhou University

In 2008, Kamiya, Takemura, and Terao studied the characteristic quasi-polynomials of integral arrangements. Subsequently, Chen and Wang investigated the arithmetic properties of the characteristic quasi-polynomial of a truncated arrangement, which is defined as a central integral arrangement restricted to a rational subspace. In this talk, we extend Chen and Wang' s results to the affine case. Specifically, we study the arithmetic properties of the characteristic quasi-polynomial of a truncated arrangement, which is an affine integral arrangement restricted to a rational affine space. This work is a joint work with Ying Cao.

Bivariate Chow polynomials for ordered matroids

Alice L.L. Gao Northwestern Polytechnical University

The real-rootedness of Hilbert–Poincaré series for Chow rings of geometric lattices was independently conjectured by Huh–Stevens and Ferroni–Schröter. Ferroni, Matherne, and Vecchi further generalized this conjecture to Chow polynomials over Cohen–Macaulay posets. In this talk, we introduce a bivariate Chow polynomial for ordered matroids and prove that such polynomials possess coefficient nonnegativity, symmetry, and unimodality. We also show that the explicit formula for uniform matroids coincides with the original Chow polynomial for q-uniform matroids. Furthermore, we establish the real-rootedness of these bivariate Chow polynomials for uniform matroids.

Bivariate Chow polynomials for ordered matroids

Recursive formulas for the polymatroid Tutte polynomial

Xiaxia Guan Taiyuan University of Technology

The Tutte polynomial is a significant invariant of graphs and matroids. In 2022, Bernardi, Kalman and Postnikov [Adv. Math. 402 (2022) 108355.] introduced the polymatroid Tutte polynomial by generalizing the Tutte polynomial from matroids to polymatroids. In this talk, we shall present two recursive formulas for the polymatroid Tutte polynomial, based on the observation that certain natural "slices" of polymatroids are again polymatroids. This is a joint work with Xian' an Jin and Tamas Kalman.

Combinatorial Algebraic Geometry: Polytopes, Fans, and Toric Varieties

Tao Gui Peking University

Combinatorial algebraic geometry (and combinatorial Hodge theory) has emerged as a highly active field of research in recent years. The area draws from three seemingly distinct origins: polytopes, Coxeter groups, and matroids. The central philosophy is that even when the actual geometric varieties do not exist, one can still develop a good cohomology theory by imagining their existence and recovering the beautiful properties of classical Hodge theory. In this talk, I will provide a motivated introduction to this field, focusing on the classical interplay between polytopes, fans, and toric varieties. After

reviewing McMullen's celebrated g-conjecture (1971) and Stanley' s proof of the necessity, I will present our recent results on a graded character formula for actions of any finite group on the Artinian reduction of the Stanley–Reisner ring of any complete simplicial fan, which is given by an equivariant version of the classical h-polynomial. As an application, we prove that the Poincaré polynomial of the quotient of a compact toric orbifold by any finite reflection group is equal to the Poincaré polynomial of another compact toric orbifold. I will end with an open question of Stanley on whether the total (ungraded) representation carried by the cohomology of a compact toric orbifold under a proper action of a finite group is a permutation representation.

Monomial expansion of modified Macdonald polynomials

Emma Yu Jin Xiamen University

We discover a family of sixteen statistics on fillings of an arbitrary Young diagram and prove new combinatorial formulas for modified Macdonald polynomials. Building on these new formulas, we establish four compact expressions for the modified Macdonald polynomials, which enable us to derive four explicit formulas for the monomial expansion of the modified Macdonald polynomials. One of them coincides with the formula given by Garbali and Wheeler (2020). This talk is based on joint work with Xiaowei Lin.

The peak algebra in noncommuting variables

Shu Xiao Li Shandong University

We will give an overview on the theory of the peak algebra, and introduce an analogue in noncommuting variables. We showed that our generalizations possess many properties analogous to their classical counterparts. We will discuss some recent progress on its algebraic and combinatorial properties. This is based on joint work with Farid Aliniaeifard.

The intersection of enumerative combinatorics with coding theory, mathematical biology, and number theory

Zhicong Lin Shandong University

I will present three instances of the intersection of Enumerative Combinatorics with Coding Theory, Mathematical Biology, and Number Theory:

- Constrained Coding Bounds via Goulden–Jackson Cluster Theorem.
- Proof of a conjecture on tree-child networks proposed by Pons and Batle via Young tableaux with walls.
- Arithmetic of tangent numbers and binomial-secant numbers via combinatorial identities and the Kummer p-adic Theorem.

Hyperplane arrangement, tope graph and magnitude

Ye Liu Xi'an Jiaotong-Liverpool University

Magnitude is a cardinality-like invariant of metric spaces or enriched categories measuring the effective size. Its categorification, the magnitude homology, is a more powerful invariant. For a real hyperplane arrangement, or more generally, an oriented matroid, the tope graph encapsulates considerable amount of information. Since tope graphs are equipped with the shortest path metric, we feed them to the magnitude and magnitude homology machinery to derive new invariants of real hyperplane arrangements. We prove some structural results of the magnitude of arrangements, including reciprocity, palindromic numerator and denominator, also a face decomposition formula. For magnitude homology of arrangements, we give combinatorial descriptions in small length and prove that tope graphs are diagonal if and only if the arrangement is Boolean. We present a face decomposition of magnitude homology, using which we obtain a combinatorial formula of diagonal magnitude Betti numbers. Some results are based on joint work with Junnosuke Koizumi.

Maximal bumpless pipedreams of double Grothendieck polynomials

Sophie C.C. Sun Tianjin University of Finance and Economics

Pipedreams and bumpless pipedreams are two combinatorial models that compute double Grothendieck polynomials. While studying matrix Schubert varieties, Pechenik, Speyer, and Weigandt defined a permutation statistic rajcode( ) that captures the leading monomial of the top-degree components of a Grothendieck polynomial. Combinatorially, their result implies that there exists a unique pipedream (or bumpless pipedream) with row weight rajcode(w) and column weight rajcode(w^{-1}). An explicit construction of such

a pipedream was subsequently given by Chou and Yu. In this paper, we resolve the bumpless pipedream version of this problem by providing a recursive algorithm.

Graph Positivities

David G.L. Wang Beijing Institute of Technology

This talk provides a brief introduction to the study of e-positivity and Schur positivity of chromatic symmetric functions of graphs. Two major conjectures in this area are the Stanley–Stembridge conjecture (1993) and the Stanley–Gasharov conjecture (1998), both of which concern the positivity of chromatic symmetric functions for certain graph classes. Although the former was proved by Hikita in 2024, the latter remains largely open, with only partial progress to date. I will review the motivation, recent progress, related conjectures, and some of our own work on these problems.

Convolution-type Identity of Geometric Semilattice

Suijie Wang Hunan University

Research on convolution-type identities can be traced back to Tutte-theoretic decompositions in the work of Kook, Reiner, and Stanton on matroid Tutte polynomials. Subsequently, Kung established a matroid-based multiplication formula for central arrangements, while Wang extended convolution formulas to locally finite posets. In a different direction, Zaslavsky introduced a 1-specialized decomposition for real affine arrangements, which was later generalized to geometric semilattices by Southerland, Southern, and Zhou through the centralization construction. Building on Southerland-Southern-Zhou' s semimatroid convolution formula，this paper further generalizes it to a bivariate convolution formula of the characteristic polynomial for geometric semilattices. When specialized to hyperplane arrangements, the formula admits valuation-theoretic explanations for special cases

有理凸多胞形格点计数的组合方法及其进展

Guoce Xin Capital Normal University

有理凸多胞形中的格点计数问题，即线性丢番图方程组的整数解计数，是代数组合学与计算几何中的一个重要课题，也是本报告的核心内容。

在计算几何方向，Barvinok 于 1994 年提出了在固定维数下计算格点数的多项式时间算法。该算法随后由 De Loera 等人于 2004 年以 C 语言软件包 LattE 实现，并经过多年发展，成为该领域广泛使用的主流工具。然而，高维情况下的格点计数仍存在计算挑战。

在代数组合学方向，MacMahon 分拆分析已成为处理线性丢番图问题的常用框架。其核心是计算 Elliott–有理函数的常数项，其中包含有理凸多胞形格点计数与 Ehrhart 级数计算作为特例，与计算几何方向有密切交叉。

本报告将介绍我们团队在这一交叉领域的工作：我们将几何中的“锥体”概念引入代数组合框架，并通过以下三步系统处理上述核心问题：i) 分解为单纯锥；ii) 进一步分解为幺模单纯锥；iii) 消除松弛变量。我们将汇报团队在这三个步骤中分别取得的研究进展。新算法与当前主流工具 LattE 相比表现出明显优势，并且我们还发现了新的多胞形类，可在多项式时间内完成格点计数。

Positivity of Double Schubert Polynomials

Rui Xiong University of Ottawa

In this talk, I will discuss several positivity phenomena for double Schubert polynomials and explain how they follow from a refined version of Graham' s positivity theorem. These include the monomial positivity of Schubert polynomials, the positivity of the localization formula, the positivity of equivariant Schubert constants, and the positivity of triple Schubert constants. The last result, known as Samuel' s conjecture, implies Kirillov' s conjecture on the positivity of skew divided difference operators applied to Schubert polynomials. I will also present some combinatorial motivations that were not included in our paper. This is joint work with Yibo Gao.

Renormalization of quasisymmetric functions and its applications to multiple zeta values

Houyi Yu Southwest University

In this talk, we report our recent progress on applying the method of renormalization to realize weak quasisymmetric functions as power series. The specialization of the power series not only recovers the MZVs with nonnegative arguments, but also extends the stuffle relation of convergent MZVs to analytically continued MZVs. This is a joint work with Li Guo and Bin Zhang.

Motivic Chern classes of open projected Richardson varieties and of affine Schubert cells

Changlong Zhong State University of New York at Albany

Open projected Richardson varieties are indexed by pairs of Weyl group elements (u,w) with u <= w and w a minimal length representative. It is known that there is an embedding of these elements into the extended affine Weyl group, and there is also a geometric isomorphism behind this combinatorial construction. One can then consider the cohomology/K-theory classes. For example, He-Lam proved that the cohomology/K-theory classes of closed projected Richardson varieties coincide with opposite Schubert class in the affine Grassmannian, and Fan-Guo-Su-Xiong proved that the Segre-MacPherson classes of open projected Richardson varieties coincide with Segre-MacPherson classes of opposite Schubert cells. In this talk, I will talk about the generalization of these results into motivic Chern classes.

A proof of the multi-component q-Baker–Forrester conjecture

Yue Zhou Central South University

The Selberg integral, an n-dimensional generalization of the Euler beta integral, plays a central role in random matrix theory, Calogero–Sutherland quantum many body systems, Knizhnik–Zamolodchikov equations, and multivariable orthogonal polynomial theory. The Selberg integral is known to be equivalent to the Morris constant term identity. In 1998, Baker and Forrester conjectured a $(p+1)$-component generalization of the q-Morris identity. It in turn yields a generalization of the Selberg integral. The $p = 1$ case of Baker and Forrester's conjecture was proved by Károlyi, Nagy, Petrov and Volkov in 2015.

In this talk, we present our proof of the $(p+1)$-component q-Baker–Forrester conjecture, thereby settling this 27-year-old conjecture.