Singularities of Hamilton–Jacobi Equations and Related Topics

Geometric, Control and Stochastic Aspects

28 June – 4 July, 2026

Tianyuan Mathematics Research Center, Kunming, China

Tianyuan Mathematics Research Center — Advanced Workshop

天元數學國際交流中心

Tianyuan Mathematics Research Center

Last updated: June 27, 2026

About the Workshop

This advanced workshop focuses on the **singularity theory of Hamilton–Jacobi equations**—a central mathematical challenge with breakthrough applications across the calculus of variations, optimal control, differential geometry, nonlinear partial differential equations, and optimal transport. The workshop brings together leading international experts to analyse the structure of singularities from multiple disciplinary perspectives, to foster synergy between fundamental theory and computational methods, and to chart new directions at the frontier of modern mathematics and mathematical physics.

Organizers:

- Jianlu Zhang — AMSS, Chinese Academy of Sciences, Beijing
- Wei Cheng — Nanjing University
- Konstantin Khanin — BIMSA, Beijing

Venue: Tianyuan Mathematics Research Center, Kunming, Yunnan, China

Dates: 28 June – 4 July, 2026

Website: http://maths.nju.edu.cn/~chengwei/TYMC/

Programme

All talks take place in the main lecture hall. Each talk is 50 minutes including discussion.

Day	Morning (9:30–12:00)	Afternoon (Mon 15:00, Tue/Thu 14:30)
Mon 6/29	Cannarsa, Tran	Wang KZ, Mendico
Tue 6/30	Rassoul-Agha, Li L	Mitake, Hong, Liu GY
Wednesday 1 July — 9:00 start		
Wed 7/1	Khanin, Yu YF, Cheng	Excursion
Thu 7/2	Sorrentino, Chen	Jin L, Han, Qi
Fri 7/3	Iturriaga, Wang ZB	—

Wednesday 9:00 start. Friday ends at 11:50.

Monday, 29 June 2026

09:20–09:30	Opening remarks
	Morning session — Chair: Jianlu Zhang
09:30–10:20	**Piermarco Cannarsa** *Long-Time Behavior of Generalized Gradient Flows for Critical HJ Equations*
10:20–10:50	*Coffee break*
10:50–11:40	**Hung V. Tran** *Quantification of Ergodicity for HJ Equations in a Dynamic Random Environment*
	Afternoon session — Chair: Wei Cheng
15:00–15:50	**Kaizhi Wang** *New Dynamical Phenomena for the HJ Equation*
15:50–16:10	*Coffee break*
16:10–17:00	**Cristian Mendico** *Ergodicity in Optimal Control: A Weak KAM Approach*

Tuesday, 30 June 2026

	Morning session — Chair: Konstantin Khanin
09:30–10:20	**Firas Rassoul-Agha** *Shocks, Instability, and the Twenty Networks of Infinite Geodesics in the Directed Landscape*
10:20–10:50	*Coffee break*
10:50–11:40	**Liying Li** *Stationary Solutions for Stochastic Burgers Equation on the Real Line*
	Afternoon session — Chair: Hung V. Tran
14:30–15:20	**Hiroyoshi Mitake** *Optimal Semiconcavity with Fractional Modulus for HJ Equations with Neumann Boundary Conditions*
15:20–16:10	**Jiahui Hong** *Generalized Hamiltonian Gradient Flow and Propagation of Singularities*
16:10–16:30	*Coffee break*
16:30–17:20	**Gengyu Liu** *Comparison Principle of General HJ Equations and Applications*

Wednesday, 1 July 2026

Morning session — Chair: Andrei Sobolevski	
09:00–09:50	**Konstantin Khanin** *Open transport problems related to the Hamilton–Jacobi equation*
09:50–10:40	**Yifeng Yu** *Nonexistence of Vanishing-Viscosity Limits for Mechanical Hamiltonian Ergodic Problems*
10:40–11:10	*Coffee break*
11:10–12:00	**Wei Cheng** *Some New Problems from HJ Equations: Dynamics, Transport, Fluids, and Randomness*
12:00–14:00	*Lunch break*
14:00–17:30	**Free discussion / Excursion**

Thursday, 2 July 2026

Morning session — Chair: Yifeng Yu	
09:30–10:20	**Alfonso Sorrentino** *Stochastic Homogenization of the HJ Equation on Manifolds*
10:20–10:50	*Coffee break*
10:50–11:40	**Qinbo Chen** *A New Selection Problem for Degenerate Viscous HJ Equations*
Afternoon session — Chair: Piermarco Cannarsa	
14:30–15:20	**Liang Jin** *Inhomogeneous Discounted HJ Equations: A Dynamical Approach*
15:20–16:10	**Yuxi Han** *Quantitative Homogenization of First-Order Convex HJ Equations in Wasserstein Space*
16:10–16:30	*Coffee break*
16:30–17:20	**Junkai Qi** *On Convergence Rates for Vanishing Viscosity of Degenerate HJ Equations*

Friday, 3 July 2026

Morning session — Chair: Alfonso Sorrentino	
09:30–10:20	**Renato Iturriaga** *Instability and Scattering for Homographic Motions*
10:20–10:50	*Coffee break*
10:50–11:30	**Zibo Wang** *On the Vanishing Viscosity Limit of HJ Equations*
11:30–11:50	**Free discussion**

Abstracts

Piermarco Cannarsa

University of Rome "Tor Vergata", Italy

Long-Time Behavior of Generalized Gradient Flows for Critical Hamilton–Jacobi Equations

We investigate the long-time behavior of generalized gradient flows associated with semiconcave viscosity solutions of critical Hamilton–Jacobi equations for mechanical Hamiltonians on the flat torus. We first show that the critical set—defined as the set of points where the superdifferential contains the origin—acts as an approximate attractor for the flow. In the case of solutions to the critical equation, the critical set naturally decomposes into regular and singular components, and we establish a dichotomy describing the asymptotic behavior of trajectories, depending on which part is approached. Our approach relies on the introduction of occupational measures, which provide a measure-theoretic description of the asymptotic distribution of trajectories and yield new invariant-measure characterizations of the flow. A key ingredient is a complete proof of the global invariance of the singular set under the generalized gradient flow, filling a gap in previous literature. These results highlight a robust connection between weak KAM theory, nonsmooth analysis, and dynamical systems, and open the way to further developments for more general Hamiltonians and non-stationary problems.
Joint work with Paolo Albano (Univ. Bologna), Wei Cheng (Nanjing Univ.), and Cristian Mendico (Univ. Bourgogne Europe, Dijon).

Qinbo Chen

Nanjing University, China

A New Selection Problem for Degenerate Viscous Hamilton–Jacobi Equations

This talk concerns the selection problem for degenerate viscous Hamilton–Jacobi equations with convex Hamiltonians $H(x, Du) = \mathrm{tr}(A(x)D^2u) + c_H$ on $\mathbb{T}^n$, where the diffusion matrix $A(x)$ is nonnegative definite and c_H is the ergodic constant. We propose an approximation procedure that combines a nonlinear discounted approximation with a small potential perturbation. We then investigate how their simultaneous effects determine the asymptotic selection of viscosity solutions of the associated ergodic problem. The main ingredients of our analysis are generalized Mather measures and the nonlinear adjoint method.

Wei Cheng

Nanjing University, China

Some New Problems from Hamilton–Jacobi Equations: Dynamics, Transport, Fluids, and Randomness

To stimulate interdisciplinary collaboration, this talk will explore several new and open research directions. We will discuss the relationship between singularity theory for viscosity solutions of Hamilton–Jacobi equations and irreversible phenomena in various areas of mathematical physics. The potential intrinsic commonality of these irreversible phenomena will be discussed, with the goal of offering new perspectives on several topics, including some classical problems. These ideas are based on our research on singularity theory for viscosity solutions of Hamilton–Jacobi equations over the past decade, especially the recent progress on generalized Hamiltonian gradient flows in the last two years.

Joint work with Piermarco Cannarsa (Rome "Tor Vergata"), Jiahui Hong (Nanjing University), Kaizhi Wang (Shanghai Jiao Tong University), and students Tianqi Shi, Wenxue Wei, Zhixiang Zhu.

Yuxi Han

Purdue University, USA

Quantitative Homogenization of First-Order Convex Hamilton–Jacobi Equations in Wasserstein Space

Hamilton–Jacobi equations naturally arise in mean field control problems. For interacting agents moving in a heterogeneous landscape, as the scale of the heterogeneities tends to zero, the value function of the mean field control problem is expected to converge uniformly to the solution of an effective Hamilton–Jacobi equation in Wasserstein space, whose Hamiltonian is characterized through an appropriate cell problem. In this talk, we establish quantitative rates of convergence for the homogenization of first-order convex Hamilton–Jacobi equations in Wasserstein space and extend the analysis to dynamic optimal transport problems.

Joint work with Zhiyan Ding, Ibrahim Ekren, and Antonios Zitridis.

Jiahui Hong

Nanjing University of Aeronautics & Astronautics, China

Generalized Hamiltonian Gradient Flow and Propagation of Singularities in Hamilton–Jacobi Equations and Optimal Transport

In this talk, we begin with the problem of propagation of singularities for Hamilton–Jacobi equations. We introduce a novel theory of generalized Hamiltonian gradient flow for any semiconcave function and Hamiltonian. For weak KAM solutions, we give a result on the global propagation of cut points. We also obtain the continuity equation along generalized Hamiltonian gradient flow which clarifies the mass transport nature in the problem of propagation of singularities. Using the lifting method, we study the invariant measure of generalized Hamiltonian gradient flow and its relation with Mather measure. In the framework of optimal transport, we investigate the singularities of potential energy functionals associated with semiconcave functions and weak KAM solutions in the Borel probability measure space and their propagation properties.

Renato Iturriaga

CIMAT, Guanajuato, Mexico

Instability and Scattering for Homographic Motions

In the n-body problem framework, we use a decomposition of the Jacobi equation to study homographic solutions along ellipses or hyperbolas, based on a strong minimizing central configuration. In the case of ellipses, we show instability, and in the hyperbolic case, a local scattering result.

Kaizhi Wang

Shanghai Jiao Tong University, China

New Dynamical Phenomena for the Hamilton–Jacobi Equation

The weak KAM theorem tells us that the dynamical behavior of the semiflow generated by the Hamilton–Jacobi equation, which originates from classical mechanics, is simple. However, the dynamical behavior of the semiflow generated by the contact Hamilton–Jacobi equation, which has profound backgrounds in fields such as thermodynamics, is completely different. This talk will cover topics related to the (in)stability, the existence and multiplicity of periodic and quasi-periodic solutions, as well as chaotic phenomena, concerning the contact Hamilton–Jacobi equation.

Liang Jin

Nanjing University of Science & Technology, China

Inhomogeneous Discounted Hamilton–Jacobi Equations: A Dynamical Approach

In this talk, we shall introduce our recent results concerning the inhomogeneous discounted Hamilton–Jacobi equation $\lambda(x)u+H(x,d_xu)=c$ and stability theory of Hamilton–Jacobi equations. Our discussion will cover: (1) the definition of the critical value and the solvability of the ergodic problem; (2) the relationship between the critical value and the existence of asymptotically stable solutions; (3) the long-time behavior and convergence rate of the solution semigroup; (4) a dynamical criterion for the critical value.

Joint work with Jun Yan (Fudan University) and Kai Zhao (Tianjin University).

Junkai Qi

Nanjing University, China

On Convergence Rates for Vanishing Viscosity of Degenerate Hamilton–Jacobi Equations

We study convergence rates for the vanishing viscosity approximation of degenerate viscous Hamilton–Jacobi equations with convex Hamiltonians and spatially dependent diffusion. Using the adjoint method combined with entropy estimates, we establish the sharp rate $O(\varepsilon|\log\varepsilon|)$.

Konstantin Khanin

BIMSA, Beijing, China

Open Transport Problems Related to the Hamilton–Jacobi Equation

In this talk, I will discuss several open transport problems related to the Hamilton–Jacobi equation across different settings. First, I will address the well-known open problem of defining dynamics for characteristics beyond the moment of shock formation. Next, I plan to explore various approaches to the "resolution of singularities." Finally, time permitting, I will discuss transport problems in the randomly forced Hamilton–Jacobi equation, highlighting their close connection to Kardar–Parisi–Zhang (KPZ) universality.

Liying Li

Southern University of Science and Technology (SUSTech), Shenzhen, China

Stationary Solutions for Stochastic Burgers Equation on the Real Line

We study the viscous Burgers equation on the real line with additive white-in-time random force. In the case of space-homogeneous noise, under mild assumptions we establish the existence of a unique ergodic solution for every fixed average velocity. The analysis utilizes the directed polymer models derived from the Feynman–Kac representation of the Burgers solution, where the question of unique ergodicity can be reformulated in terms of the existence and uniqueness of the infinite-volume polymer Gibbs measures. Furthermore, the inviscid limit holds for these ergodic solutions, and correspondingly, the polymer measures in the zero-temperature limit converge to delta measures concentrated on infinite minimizers. We will discuss briefly the generalizations and challenges in higher dimensions and for general non-quadratic Hamiltonians, as well as certain KPZ universality problems related to the polymer measures. In the case of L^2 noise, we present a polynomial mixing result using Foiaş–Prodi type estimates, which holds for general conservation laws as well.

Gengyu Liu

AMSS, CAS, Beijing, China

Comparison Principle of General Hamilton–Jacobi Equations and Applications

In this talk, we investigate contact Hamilton–Jacobi equations $H(x, du, u) = c$ and $H(x, du, u) = c + \Delta u$ under the condition that the contact direction is not strictly positive definite. Correspondingly, we describe the structure of $\mathcal{C}$ containing all the $c \in \mathbb{R}$ that makes the equations solvable and establish general comparison principles. As applications, we employ these comparison principles to obtain quantitative homogenization results.

Cristian Mendico

Université Bourgogne Europe, Dijon, France

Ergodicity in Optimal Control: A Weak KAM Approach

This talk is devoted to ergodic phenomena in optimal control from the viewpoint of weak KAM theory. I will present recent developments on Aubry–Mather and weak KAM theory in the sub-Riemannian setting and for control-affine systems with drift. A central motivation comes from acceleration-type dynamics, where viscosity solutions of the ergodic Hamilton–Jacobi equation may fail to be continuous. This phenomenon highlights a genuinely nonclassical ergodic behavior and calls for new tools beyond the standard Tonelli framework. I will discuss the construction of critical solutions and the role of Aubry-type sets in describing the asymptotic structure of minimizing trajectories (possibly singular). Special emphasis will be given to the geometric features induced by nonholonomic constraints and drift terms. I will then explain how these ideas lead to a unified weak KAM approach for a broad class of control systems.

Hiroyoshi Mitake

Waseda University, Tokyo, Japan

Optimal Semiconcavity with Fractional Modulus for Hamilton–Jacobi Equations with Neumann Boundary Conditions

We study the semiconcavity property of viscosity solutions to Hamilton–Jacobi equations with Neumann boundary conditions. Unlike the state-constraint case, minimizing trajectories associated with the Neumann problem may fail to be C^1, so the classical approach based on the regularity of minimizers is no longer available. To overcome this difficulty, we introduce a comparison argument between the constrained action associated with the Skorokhod problem and the unconstrained action, avoiding any use of higher regularity of minimizing trajectories. Under a structural decomposition assumption on the Hamiltonian at the boundary, we establish the global semiconcavity with a fractional modulus and show the optimality of the fractional exponent.
Joint work with Panrui Ni (Waseda Univ.).

Firas Rassoul-Agha

University of Utah, USA

Shocks, Instability, and the Twenty Networks of Infinite Geodesics in the Directed Landscape

For stochastic Hamilton–Jacobi equations, instability points are where two eternal solutions with the same asymptotic velocity differ, while shocks are where the velocity field is discontinuous. We analyze these structures for the KPZ fixed point, the central object of the KPZ universality class. We describe the geometry of the instability region, show that it can be determined by the shock structures of the two eternal solutions, and obtain a complete classification of semi-infinite geodesics/characteristics. We also identify the Martin boundary and its minimal part with horofunctions and Busemann functions, respectively. As a consequence of instability, the Martin boundary is strictly larger than its minimal part.

Joint work with Mikhail Sweeney.

Alfonso Sorrentino

University of Rome "Tor Vergata", Italy

Stochastic Homogenization of the Hamilton–Jacobi Equation on Manifolds

In this talk, I will present a stochastic homogenization result for first-order Hamilton–Jacobi equations on Riemannian manifolds within a stationary ergodic random environment. The setting involves a finitely generated abelian group of isometries acting freely and cocompactly on the manifold, together with a family of Hamiltonians parametrized by an ergodic probability space. Under standard assumptions — including strict convexity and superlinear coercivity in the momentum variable — I will show that as the scaling parameter tends to zero, the viscosity solutions to the rescaled Hamilton–Jacobi equation converge almost surely and locally uniformly to the solution of a deterministic homogenized equation posed on a Euclidean space of dimension d, where d corresponds to the rank of the acting group.

Joint work with Marco Pozza.

Hung V. Tran

University of Wisconsin–Madison, USA

Quantification of Ergodicity for Hamilton–Jacobi Equations in a Dynamic Random Environment

We study quantitative large-time averages for Hamilton–Jacobi equations in a dynamic random environment that is stationary ergodic and has unit-range dependence in time. Our motivation comes from stochastic growth models related to the tensionless (inviscid) KPZ equation, which can be formulated as Hamilton–Jacobi equations with random forcing. First, I will describe some important open questions in this direction. Second, I will describe our main results on convergence rates with exponent 1/2 for the large-time averages of both the solutions and the associated metric problem toward their ergodic limits. I will then focus on a new almost-Lipschitz regularity theory for the metric problem.

Yifeng Yu

University of California, Irvine, USA

Nonexistence of Vanishing-Viscosity Limits for Mechanical Hamiltonian Ergodic Problems

In this talk, I will discuss a recent counterexample to the vanishing-viscosity selection problem for mechanical Hamiltonian ergodic equations. Consider the viscous cell problem

$$\tfrac{1}{2}|D\phi^\varepsilon|^2 + F(x) - \varepsilon\Delta\phi^\varepsilon = c(\varepsilon) \qquad \text{on } \mathbb{T}^n,$$

with the normalization $\phi^\varepsilon(0) = 0$. A long-standing question asks whether, as $\varepsilon \to 0$, the solutions ϕ^ε always converge to a uniquely selected viscosity solution of the corresponding inviscid cell problem. We give a negative answer to this question. In particular, we construct a one-dimensional mechanical Hamiltonian with $F \in C^3(\mathbb{T})$, and more generally examples in every finite regularity class C^k, for which the full vanishing-viscosity limit does not exist. This disproves the expected selection principle and answers a problem proposed by Jauslin, Kreiss, and Moser in 1999.

Joint work with Ziran Liu and Hung V. Tran.

Zibo Wang

AMSS, Chinese Academy of Sciences, Beijing, China

On the Vanishing Viscosity Limit of Hamilton–Jacobi Equations

In this talk, we try to develop a unified framework for the vanishing viscosity limit of Hamilton–Jacobi equations within the class of Tonelli Hamiltonians. We also provide quantitative estimates on the rate of convergence.

Joint work with Professor Jianlu Zhang (AMSS, CAS).

Participants

Name	Affiliation	Email
Piermarco Cannarsa	Univ. Rome "Tor Vergata", Italy	cannarsa@axp.mat.uniroma2.it
Qinbo Chen	Nanjing University, China	qinbochen@nju.edu.cn
Wei Cheng	Nanjing University, China	chengwei@nju.edu.cn
Yuxi Han	Purdue University, USA	han891@purdue.edu
Jiahui Hong	NUAA, Nanjing, China	hongjiahui@nuaa.edu.cn
Renato Iturriaga	CIMAT, Guanajuato, Mexico	renato@cimat.mx
Liang Jin	NJUST, Nanjing, China	jl@njust.edu.cn
Konstantin Khanin	BIMSA, Beijing, China	khanin@math.toronto.edu
Liying Li	SUSTech, Shenzhen, China	lily@sustech.edu.cn
Gengyu Liu	AMSS, CAS, Beijing, China	[??]
Ziran Liu	SIMIS, Shanghai, China	zliu@simis.cn
Cristian Mendico	Univ. Bourgogne Europe, Dijon, France	Cristian.Mendico@u-bourgogne.fr
Hiroyoshi Mitake	Waseda University, Tokyo, Japan	mitake@waseda.jp
Junkai Qi	Nanjing University, China	junkaiqinju@outlook.com
Firas Rassoul-Agha	University of Utah, USA	firas@math.utah.edu
Andrei Sobolevski	HSE University, Moscow, Russia	sobolevskii.an@mipt.ru
Alfonso Sorrentino	Univ. Rome "Tor Vergata", Italy	sorrentino@mat.uniroma2.it
Hung V. Tran	Univ. Wisconsin–Madison, USA	hung@math.wisc.edu
Kaizhi Wang	Shanghai Jiao Tong University, China	kzwang@sjtu.edu.cn
Zibo Wang	AMSS, CAS, Beijing, China	zibowang@amss.ac.cn
Yifeng Yu	UC Irvine, USA	yifengy@uci.edu
Jianlu Zhang	AMSS, CAS, Beijing, China	jzhang87@amss.ac.cn