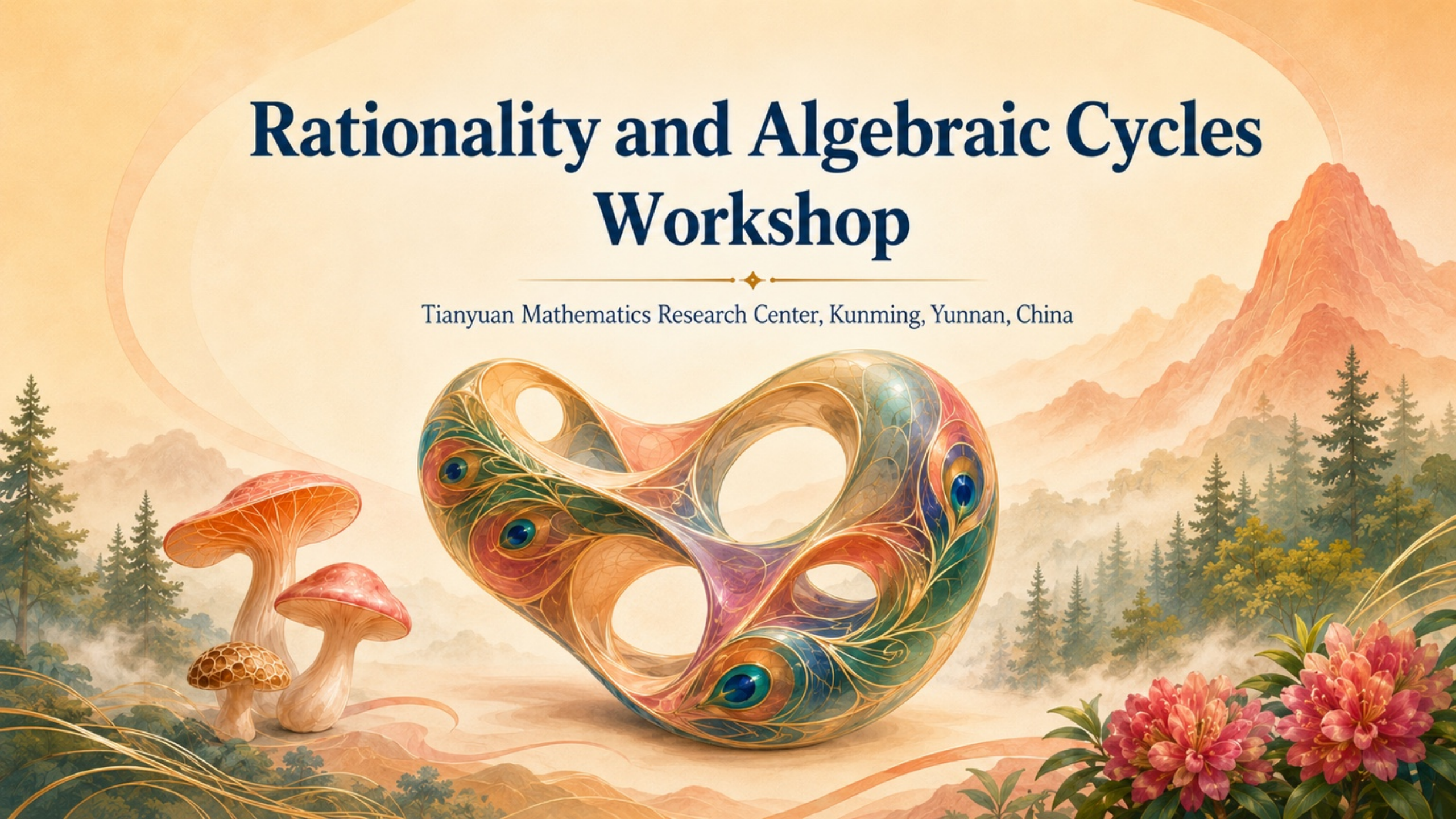


Rationality and Algebraic Cycles Workshop

Tianyuan Mathematics Research Center, Kunming, Yunnan, China



Schedule

Monday 24 Aug	Tuesday 25 Aug	Wednesday 26 Aug	Thursday 27 Aug	Friday 28 Aug
9:00-10:00 Lena Ji Rationality of real conic bundle threefolds	9:00-10:00 Zhi Jiang A family of minimal surfaces of general type with $K^2 = 4$ and $p_g = q = 1$	9:00-10:00 Evgeny Shinder Tate-stable equivalence of Chow motives	9:00-10:00 Valery Lunts Quantum elliptic curve	9:00-10:00 Jie Xu Fibration question and strong approximation for the intersection of two quadrics
10:30-11:30 Margot Bruneaux Comb smoothing and local triviality of homogeneous spaces over a relative curve	10:30-11:30 Zhiyuan Li A survey on unirationality of supersingular surfaces	10:30-11:30 Jan Lange On zero-cycles of varieties over Laurent fields	10:30-11:30 Fumiaki Suzuki Motivic classes of Fano schemes of lines on intersections of two quadrics	10:30-11:30 Lu Qi Boundedness in general type MMP and local K-stability
Afternoon Open Problem Session	14:00-15:00 Ting Gong Generalized Theta group in algebraic geometry	14:00-15:00 Qizheng Yin On the ring structure of the cohomology of compactified Jacobians	14:00-15:00 Weite Pi Brauer groups of smooth loci in linear systems and Tate-Shafarevich twists for plane curves	Afternoon Discussion Session
	15:30-16:30 Federico Scavia Potential vanishing of degree 3 unramified cohomology over finite fields	15:15-16:15 Yidi Wang Local-global principles on stacky curves and its application in solving generalized Fermat equations	15:30-16:30 Lin Zhou On the infinite generation of morphic and motivic cohomology	
		16:30-17:30 Zheng Xu On the rational connectedness of globally $\pm$ -regular varieties		

Participants

Organizers and speakers · sorted by last name

ORGANIZERS

Rajesh Kulkarni

Michigan State University

Zhiyu Tian

Peking University

Shitan Xu

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PARTICIPANTS AND SPEAKERS

Margot Bruneaux

Claude Bernard University Lyon 1

Ting Gong

University of Washington

Lena Ji (online)

University of Illinois Urbana-Champaign

Zhi Jiang

Fudan University

Jan Lange

Leibniz University Hannover

Zhiyuan Li

Fudan University

Valery Lunts (online)

Indiana University Bloomington

Weite Pi

University of Bonn

Lu Qi

East China Normal University

Federico Scavia

CNRS and Sorbonne Paris North University

Evgeny Shinder

University of Sheffield

Fumiaki Suzuki

Peking University

Yidi Wang

University of Western Ontario

Jie Xu

Peking University

Zheng Xu

Peking University

Qizheng Yin

Peking University

Lin Zhou

Leibniz University Hannover

• Lena Ji

University of Illinois Urbana-Champaign

TITLE

Rationality of real conic bundle threefolds

ABSTRACT

Over algebraically closed fields, conic bundles over P^2 are a class of threefolds whose rationality has been well studied. Over non-closed fields, however, much less is known. In this talk, we investigate the rationality problem over the real numbers, where intermediate Jacobians and topological obstructions can be used to shed light on this problem. This is based on joint work with Sarah Frei, Soumya Sankar, Bianca Viray, and Isabel Vogt and with Mattie Ji.

• Margot Bruneaux

Claude Bernard University Lyon 1

TITLE

Comb smoothing and local triviality of homogeneous spaces over a relative curve

ABSTRACT

Let R be a Henselian local ring, let k be the residue field of R , let C be a smooth projective curve over R with geometrically connected fibers, let G be a reductive C -group with isotrivial radical torus $\text{rad}(G)$, and let $E \rightarrow C$ be a G -torsor. We show that, if either the kernel of the central isogeny $G^{\text{sc}} \times_C \text{rad}(G) \rightarrow G$ is étale over C or k is large, the Zariski-local triviality of $E_k \rightarrow C_k$ implies the Zariski-local triviality of $E \rightarrow C$. We also prove an averaged form of this result, assuming only that $\text{rad}(G)$ is isotrivial, as well as a variant for projective homogeneous spaces under no restrictions on G . As consequences, we obtain a local-global principle for torsors over function fields of curves over Henselian discrete valuation rings, strengthening work of Gille-Parimala-Suresh and a Henselian version of a theorem of Drinfeld-Simpson. Our proofs are geometric and rely on compactifications of torsors and on a relative and arithmetic version of the comb smoothing technique, building on work of Kollar and Graber-Harris-Starr. (conjoint work with F. Scavia).

• Zhi Jiang

Fudan University

TITLE

A family of minimal surfaces of general type with $K^2 = 4$ and $p_g = q = 1$

ABSTRACT

About 30 years ago, Catanese and Ciliberto showed that for a minimal surface S of general type with $K^2 = 3$ and $p_g = q = 1$, the general fiber of the Albanese morphism of S has genus at most 3 and, when the genus is 3, these surfaces are deformation equivalent. We showed that an analogue holds for minimal surfaces S of general type with $K^2 = 4$ and $p_g = q = 1$: the general fiber of the Albanese morphism of S has genus at most 5 and, when the genus is 5, these surfaces are deformation equivalent. This talk is based on joint work with Hsueh-Yung Lin.

• Zhiyuan Li

Fudan University

TITLE

A survey on unirationality of supersingular surfaces

ABSTRACT

This talk will survey the unirationality problem for algebraic surfaces over algebraically closed fields of positive characteristic, with a focus on supersingular surfaces. I will review the classical results and recent progress toward the conjecture that every supersingular surface (in the sense of Shioda) is unirational. In particular, I will discuss the main techniques used to establish unirationality for certain classes of supersingular $K3$ surfaces and other surfaces, as well as the current obstacles and open questions in the field.

• Ting Gong

University of Washington

TITLE

Generalized Theta group in algebraic geometry

ABSTRACT

In the 1960s, Mumford suggested the idea of theta groups on abelian varieties and subsequently, O'Neill, Sharif and Clark used this technique to solve some period index problems in both elliptic curves and Brauer classes on curves. In this talk, we introduce a generalized theta group theory on general proper geometric integral varieties and reprove some classical theorems. For example, we derive a bound for the period index problem for Brauer classes and we reprove Lichtenbaum theorem on the period index problem for genus greater than two curves, as well as Sharif's prescription of period and index on a genus greater than or equal to two curve. Finally, we prove some new splitting theorems for Brauer classes. This is a joint work with Soham Ghosh and Shitan Xu.

• Federico Scavia

CNRS and Sorbonne Paris North University

TITLE

Potential vanishing of degree 3 unramified cohomology over finite fields

ABSTRACT

Let k be a finite field of characteristic p different from 3, let $\ell > 3$ be a prime different from p , and let E/k be the Fermat cubic curve. We prove that if ℓ^5 divides $[k:F_p]$, then $H^3_{\text{nr}}(k(E^3)/k, \mathbb{Q}_{\ell}/\mathbb{Z}_{\ell}(2))$ is zero. Conditionally on the Tate conjecture over finite fields, we prove an analogous result for every smooth projective variety over any finite field of characteristic not equal to 2, and give an application to a conjecture of Colliot-Thélène on the local-global principle for zero-cycles over global function fields.

Evgeny Shinder

University of Sheffield

TITLE

Tate-stable equivalence of Chow motives

ABSTRACT

I will introduce the quotient category of Chow motives by the subcategory of Tate motives and explain some of its properties. Then I will explain nontrivial examples of Tate-stable equivalence: genus one curves, Severi-Brauer varieties and derived equivalent K_3 surfaces. This is joint work in progress with Lin and Sechin.

Jan Lange

Leibniz University Hannover

TITLE

On zero-cycles of varieties over Laurent fields

ABSTRACT

Z. Tian showed that the Chow group of degree 0 zero-cycles is trivial for proper rationally connected varieties over the Laurent field $C((t))$, if the dimension is at most 3 or if the Tate conjecture for divisors on surfaces holds over finite fields. In this talk, we discuss an unconditional partial result in that direction and some obstacles for the general case.

Qizheng Yin

Peking University

TITLE

On the ring structure of the cohomology of compactified Jacobians

ABSTRACT

We study the cohomology of compactified Jacobians of (families of) reduced curves with planar singularities, with emphasis on the ring structure. For families of curves, the cohomology ring is shown to depend on the choice of a stability condition. Using the theory of perverse sheaves and algebraic cycle techniques such as the Fourier transform, we provide a degeneration of the ring structure which is independent of the stability condition. We next focus on the compactified Jacobian of a fixed integral curve with planar singularities, and present some results and open questions regarding the cohomology ring structure. Based on joint work with Younghun Bae, Daves Maulik, and Junliang Shen.

Yidi Wang

University of Western Ontario

TITLE

Local-global principles on stacky curves and its application in solving generalized Fermat equations

ABSTRACT

The primitive solutions of certain generalized Fermat equations, i.e., Diophantine equations of the form $Ax^p + By^q = Cz^r$, can be studied as integral points on certain stacky curves. Bhargava-Poonen gave an explicit example of such a curve of genus $1/2$ violating the local-global principle for integral points. However, a general description of stacky curves failing the local-global principle is unknown. In this talk, I will discuss our work on finding the primitive solutions to equations of this form when $(p, q, r) = (2, 2, n)$ by studying local-global principles for integral points on stacky curves constructed from such equations. The talk is based on a joint paper with Juanita Duque-Rosero, Christopher Keyes, Andrew Kobin, Manami Roy, and Soumya Sankar.

• Zheng Xu

Peking University

TITLE

On the rational connectedness of globally $\pm$ -regular varieties

ABSTRACT

Globally $\pm$ -regular varieties serve as mixed-characteristic analogues of singular Fano varieties. In this talk, I will introduce the basic notions and present recent results on the rational connectedness of globally $\pm$ -regular varieties. The talk is based on my joint work with Emre Alp Ozavci, Zsolt Patakfalvi, Kevin Tucker and Joe Waldron.

Valery Lunts

Indiana University Bloomington

TITLE

Quantum elliptic curve

ABSTRACT

A complex elliptic curve E can be defined as the quotient of the analytic space C^* by a discrete action of the cyclic group $q^{\mathbb{Z}}$ for $|q| \neq 1$. We study the boundary case when $|q| = 1$, which leads to the notion of a quantum elliptic curve and a conjectural equivalence of categories that one might call a noncommutative GAGA. This is joint work in progress with Michael Larsen. arXiv: 2512.06936.

Fumiaki Suzuki

Peking University

TITLE

Motivic classes of Fano schemes of lines on intersections of two quadrics

ABSTRACT

Inspired by predictions from derived categories, Belmans et al. conjectured explicit formulas in the Grothendieck ring of varieties for the classes of the Fano schemes of r -planes on smooth complete intersections of two quadrics. In this talk, we confirm their conjecture for $r = 0$ and 1 . As an input, we give a formula for the class of the relative Fano scheme of lines of a quadric fibration over P^1 with simple degeneration. This is joint work with Lena Ji.

• Weite Pi

University of Bonn

TITLE

Brauer groups of smooth loci in linear systems and Tate-Shafarevich twists for plane curves

ABSTRACT

We study the Brauer group of smooth loci in linear systems on smooth projective varieties. For suitably ample linear systems on simply connected varieties, we prove that it is at most $\mathbb{Z}/2\mathbb{Z}$, and give a precise characterization via the universal binodal locus. Most interestingly, this Brauer group vanishes for very general K_3 surfaces or general cubic fourfolds. Building on the proof strategy, we further compute the Tate-Shafarevich group of the relative Jacobian of plane curves. This talk is based on joint work with Moritz Hartlieb.

• Lin Zhou

Leibniz University Hannover

TITLE

On the infinite generation of morphic and motivic cohomology

ABSTRACT

Since Mumford's work in the 1960s, questions on the finite generation of Chow and Griffiths groups - such as the finiteness of dimension, the size of their torsion, or their divisibility - have been a central theme in the study of algebraic cycles. Motivic cohomology and morphic cohomology naturally generalize the Chow group (cycles modulo rational equivalence) and cycles modulo algebraic equivalence. In this talk, I will show how, over an algebraically closed field whose transcendence degree over its prime field is infinite (e.g., the complex number field), one can combine Schoen's injectivity argument with Schreieder's refined unramified cohomology to construct examples of motivic and morphic cohomology groups with infinitely many torsion elements. These appear to be the first known examples exhibiting infinite torsion in motivic or morphic cohomology. Joint work with Theodosis Alexandrou.

Jie Xu

Peking University

TITLE

Fibration question and strong approximation for the intersection of two quadrics

ABSTRACT

We study strong approximation for the intersection of two affine quadrics. As an application, we prove that the fibration question has a positive answer in the case of rank four with nonsplit fibers split by quadratic extensions. This is based on joint work with Prof. Dasheng Wei and Yi Zhu.

Lu Qi

East China Normal University

TITLE

Boundedness in general type MMP and local K-stability

ABSTRACT

In this talk, I will discuss how to use local volumes from the local theory of K-stability to prove a boundedness result in the Minimal Model Program for general type varieties. Based on joint work with Jingjun Han and Ziquan Zhuang.

End of abstracts